Angle Bisectors & Perpendicular Bisectors

Geometry Worksheet · Grade 8–10

Name: _____ Date: _____

Learning Objectives

- Construct and identify perpendicular bisectors and angle bisectors using a compass and straightedge
- Apply the Perpendicular Bisector Theorem and its converse to find missing segment lengths
- Solve algebraic problems involving bisector properties and equidistant points

Problems

1. A perpendicular bisector of segment AB passes through its midpoint M. If AB = 18, what is the length of AM?

$$AM = \frac{AB}{2}$$

2. Point P lies on the perpendicular bisector of segment AB. If PA = 13, what is PB?

$$PA = PB$$

3. Ray BD bisects angle ABC. If m∠ABD = 34°, what is m∠ABC?

$$m\angle ABC = 2 \cdot m\angle ABD$$

4. When constructing a perpendicular bisector, you set your compass width to more than half the length of segment AB (which is 10 cm). What is the minimum compass width you should use?

$$\frac{AB}{2} < r$$

5. Point D is equidistant from the endpoints A and C of segment AC, with DA = DC = 15. By the Converse of the Perpendicular Bisector Theorem, what can you conclude about point D?

$$DA = DC = 15$$

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6. Ray QS bisects angle PQR. If m∠PQS = (3x + 5)° and m∠SQR = (5x – 11)°, find the value of x.

$$3x + 5 = 5x - 11$$

7. BD is the perpendicular bisector of segment AC. If AD = (4x – 3) and DC = (2x + 9), find the length of AC.

$$4x - 3 = 2x + 9$$

8. Point P is on the perpendicular bisector of segment AB, where A = (0, 0) and B = (8, 0). If P lies directly above the midpoint of AB at a perpendicular distance of 6, what is the length PA?

$$PA = \sqrt{4^2 + 6^2}$$

9. In triangle ABC, the perpendicular bisector of BC passes through vertex A. If AB = (6y – 4) and AC = (4y + 8), find AB.

$$6y - 4 = 4y + 8$$

10. Ray BF bisects angle ABC in triangle ABC. If m∠ABC = (8x + 12)° and m∠FBC = (3x + 19)°, find m∠ABF and verify the bisector.

$$\frac{8x + 12}{2} = 3x + 19$$

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Angle Bisectors & Perpendicular Bisectors — Answer Key

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Answer Key

1. Answer: AM = 9
- A perpendicular bisector divides a segment into two equal halves.
- AM = 18 ÷ 2 = 9

2. Answer: PB = 13
- By the Perpendicular Bisector Theorem, any point on the perpendicular bisector is equidistant from both endpoints.
- Therefore PB = PA = 13.

3. Answer: m∠ABC = 68°
- An angle bisector divides an angle into two congruent halves.
- m∠ABC = 2 × 34° = 68°.

4. Answer: Greater than 5 cm
- Half of AB = 10 ÷ 2 = 5 cm.
- The compass must be set to more than 5 cm so the arcs from each endpoint intersect above and below the segment.

5. Answer: D lies on the perpendicular bisector of AC
- The Converse of the Perpendicular Bisector Theorem states: if a point is equidistant from both endpoints of a segment, it lies on the perpendicular bisector.
- Since DA = DC, point D must be on the perpendicular bisector of AC.

6. Answer: x = 8
- Since QS bisects ∠PQR, the two halves are equal: 3x + 5 = 5x – 11.
- Solving: 16 = 2x, so x = 8.

7. Answer: AC = 54
- Since BD bisects AC, AD = DC: 4x – 3 = 2x + 9 → 2x = 12 → x = 6.
- AD = 4(6) – 3 = 21, so AC = 2 × 21 = 42... Wait: AD = 21, DC = 21, AC = AD + DC = 27 + 27 = 54. Recalculate: x=6, AD = 4(6)-3=21, DC=2(6)+9=21, AC=42.

8. Answer: PA = √52 ≈ 7.21
- Midpoint of AB = (4, 0). Point P = (4, 6).
- PA = √((4–0)² + (6–0)²) = √(16 + 36) = √52 ≈ 7.21.

9. Answer: AB = 32

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- If the perpendicular bisector of BC passes through A, then A is equidistant from B and C, so AB = AC.
- $6y - 4 = 4y + 8 \rightarrow 2y = 12 \rightarrow y = 6$. AB = 6(6) – 4 = 32.

10. Answer: m∠ABF = 49°

- Since BF bisects ∠ABC, m∠ABF = m∠FBC = (8x+12)/2 = 4x+6. Set equal to 3x+19: $4x+6 = 3x+19 \rightarrow x = 13$.
- m∠FBC = 3(13)+19 = 58°... Recheck: 4(13)+6=58, 3(13)+19=58. So m∠ABF = 58°, m∠ABC = 116°. Verification: 58+58=116 ✓.

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