

# Triangle Inequalities and Theorems

Geometry Worksheet · Grade 8–10

Name: \_\_\_\_\_

Date: \_\_\_\_\_

## Learning Objectives

- Apply the Comparison Property of Inequalities to triangle side lengths and angles
- Use the Exterior Angle Inequality Theorem to compare angles in a triangle
- Apply the Isosceles Triangle Theorem to prove angle relationships

## Problems

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1. In a triangle, if  $a = b + c$  and  $c > 0$ , which inequality must be true?

$$a = b + c, c > 0$$

2. Angle 1 is an exterior angle of a triangle with remote interior angles 2 and 3. If  $m\angle 2 = 55^\circ$  and  $m\angle 3 = 48^\circ$ , find  $m\angle 1$ .

$$m\angle 1 = m\angle 2 + m\angle 3$$

3. An exterior angle of a triangle measures  $112^\circ$ . One remote interior angle measures  $67^\circ$ . Find the measure of the other remote interior angle.

$$112^\circ = 67^\circ + x$$

4. In triangle ABC, exterior angle ACD is formed by extending side BC. Which two angles is  $m\angle ACD$  greater than?

$$m\angle ACD > m\angle A \text{ and } m\angle ACD > m\angle B$$

5. In isosceles triangle PQR,  $PQ \cong PR$ . What can you conclude about the base angles?

$$PQ \cong PR \Rightarrow m\angle Q = m\angle R$$

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6. In triangle XYZ, an exterior angle at vertex Z measures  $95^\circ$ . Can the remote interior angles each measure  $50^\circ$ ? Explain.

$$50^\circ + 50^\circ = 100^\circ \neq 95^\circ$$

7. In the figure, point M lies on segment AC of triangle ABC such that  $BM \cong BC$ . If  $m\angle ABM = 40^\circ$ , find  $m\angle BMC$  (the exterior angle of triangle ABM at M).

$$m\angle BMC = m\angle ABM + m\angle BAM$$

8. In triangle DEF, extend side EF to point G. Using the Exterior Angle Inequality Theorem, write two inequality statements involving the exterior angle DFG.

$$m\angle DFG > m\angle D \text{ and } m\angle DFG > m\angle E$$

9. In triangle ABC, point D is the midpoint of BC such that AD is drawn, creating triangles ABD and ACD.  $AB \cong AC$  (isosceles). Using the Isosceles Triangle Theorem and Exterior Angle Theorem, prove that  $m\angle ADB > m\angle ABD$ .

$$m\angle ADB = m\angle DAC + m\angle ACD$$

10. In triangle PQR, S is a point on QR such that PS is drawn. Given that  $PQ \cong PS$ ,  $m\angle QPR = 80^\circ$ , and  $m\angle PSR$  is an exterior angle of triangle PQS, find  $m\angle PSR$ .

$$m\angle PSR = m\angle PQS + m\angle QPS$$

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# Triangle Inequalities and Theorems — Answer Key

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## Answer Key

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### 1. Answer: $a > b$

- Since  $c > 0$ , adding  $c$  to  $b$  makes the sum strictly greater than  $b$  alone.
  - Therefore  $a = b + c > b$ .
- 

### 2. Answer: $103^\circ$

- By the Exterior Angle Theorem,  $m\angle 1 = m\angle 2 + m\angle 3$ .
  - $m\angle 1 = 55^\circ + 48^\circ = 103^\circ$ .
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### 3. Answer: $45^\circ$

- By the Exterior Angle Theorem, the exterior angle equals the sum of the two remote interior angles.
  - $x = 112^\circ - 67^\circ = 45^\circ$ .
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### 4. Answer: $m\angle ACD$ is greater than both $m\angle A$ and $m\angle B$

- By the Corollary to the Exterior Angle Theorem, an exterior angle is greater than each of its two remote interior angles.
  - Therefore  $m\angle ACD > m\angle A$  and  $m\angle ACD > m\angle B$ .
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### 5. Answer: $m\angle Q = m\angle R$ (base angles are congruent)

- By the Isosceles Triangle Theorem, if two sides of a triangle are congruent, the angles opposite those sides are congruent.
  - Therefore  $m\angle Q = m\angle R$ .
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### 6. Answer: No, because $50^\circ + 50^\circ = 100^\circ$ , not $95^\circ$

- The Exterior Angle Theorem requires the exterior angle to equal the sum of the two remote interior angles.
  - Since  $50^\circ + 50^\circ = 100^\circ \neq 95^\circ$ , this is impossible.
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### 7. Answer: Cannot be determined without $m\angle BAM$ ; if $m\angle BAM = 35^\circ$ , then $m\angle BMC = 75^\circ$

- By the Exterior Angle Theorem,  $m\angle BMC = m\angle ABM + m\angle BAM$ .
  - Substitute the given values:  $m\angle BMC = 40^\circ + m\angle BAM$ .
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### 8. Answer: $m\angle DFG > m\angle D$ and $m\angle DFG > m\angle E$

- $\angle DFG$  is an exterior angle of triangle DEF at vertex F.
  - By the Exterior Angle Inequality Theorem, it is greater than each remote interior angle:  $m\angle D$  and  $m\angle E$ .
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### 9. Answer: $m\angle ADB > m\angle ABD$ because $m\angle ADB$ is an exterior angle of triangle ACD and exceeds each remote interior angle

- Since  $AB \cong AC$ , by the Isosceles Triangle Theorem  $m\angle ABD = m\angle ACD$ .

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- $\angle ADB$  is an exterior angle of triangle  $ACD$ , so  $m\angle ADB = m\angle DAC + m\angle ACD > m\angle ACD = m\angle ABD$ .
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**10. Answer:  $m\angle PSR = 90^\circ$** 

- Since  $PQ \cong PS$ , triangle  $PQS$  is isosceles, so  $m\angle PQS = m\angle PSQ$ . The angles of triangle  $PQS$  sum to  $180^\circ$ , with  $m\angle QPS$  being part of  $m\angle QPR = 80^\circ$ .
  - If we take  $m\angle QPS = 80^\circ$  (angle at  $P$  in triangle  $PQS$ ), then  $m\angle PQS = m\angle PSQ = (180^\circ - 80^\circ)/2 = 50^\circ$ . The exterior angle  $m\angle PSR = m\angle PQS + m\angle QPS = 50^\circ + 80^\circ/\dots$  Using exterior angle theorem:  $m\angle PSR = 180^\circ - 50^\circ = \dots$  By the exterior angle theorem  $m\angle PSR = m\angle PQS + m\angle QPS = 50^\circ + 40^\circ = 90^\circ$ .
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