# The Hinge Theorem and Its Converse

Geometry Worksheet · Grade 8–10

Name: ________________________________ Date: __________________

**Learning Objectives**

- Apply the Hinge Theorem (SAS Inequality Theorem) to compare side lengths of two triangles given two pairs of congruent sides and unequal included angles
- Apply the Converse of the Hinge Theorem (SSS Inequality Theorem) to compare included angles of two triangles given two pairs of congruent sides and unequal third sides
- Solve for unknown angle measures and side lengths using the Hinge Theorem and its converse with algebraic expressions

## Problems

**1.** In triangles ABC and DEF, AB ≅ DE, BC ≅ EF, and ∠B = 50° while ∠E = 80°. Using the Hinge Theorem, which third side is longer?

$$\angle E > \angle B \Rightarrow DF > AC$$

**2.** In triangles PQR and XYZ, PQ ≅ XY, QR ≅ YZ, and ∠Q = 95° while ∠Y = 40°. Which third side is longer?

$$\angle Q > \angle Y \Rightarrow PR > XZ$$

**3.** In triangles MNO and RST, MN ≅ RS, NO ≅ ST, and MO = 12 while RT = 7. Using the Converse of the Hinge Theorem, which included angle is larger?

$$MO > RT \Rightarrow \angle N > \angle S$$

**4.** Triangle SAK is a right triangle with ∠A = 90°, and triangle YOU has ∠O = 70°, with SA ≅ YO and AK ≅ OU. Which statement must be true about sides SK and YU?

$$\angle A = 90° > \angle O = 70° \Rightarrow SK > YU$$

**5.** In triangles ABC and DEF, AB ≅ DE, AC ≅ DF, EF = 5, and BC = 6. Using the Converse of the Hinge Theorem, write an inequality comparing ∠A and ∠D.

$$BC > EF \Rightarrow \angle A > \angle D$$

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**6.** In triangles JKL and WXY, JK ≅ WX, KL ≅ XY, ∠K = 110°, and ∠X = 110°. What can you conclude about sides JL and WY?

$$\angle K = \angle X \Rightarrow JL = WY$$

**7.** In triangles ABC and DEF, AB ≅ DE, AC ≅ DF, ∠A = (3x + 10)°, ∠D = (5x – 6)°, and BC > EF. Find the range of values of x.

$$3x + 10 > 5x - 6 \Rightarrow x < 8$$

**8.** In triangles PQR and STU, PQ ≅ ST, QR ≅ TU, ∠Q = (4x – 5)°, and ∠T = (2x + 15)°. If PR < SU, find the range of values of x, given that all angles must be positive and less than 180°.

$$4x - 5 < 2x + 15 \Rightarrow x < 10$$

**9.** In two triangles, the two pairs of congruent sides each measure 8 cm and 13 cm. The included angle in the first triangle is 120°. If the third side of the second triangle measures 15 cm, use the Hinge Theorem to determine whether the included angle of the second triangle is greater than, less than, or equal to 120°.

$$c^2 = 8^2 + 13^2 - 2(8)(13)\cos(120°) = 337$$

**10.** In triangles ABC and DEF, AB ≅ DE = 10, AC ≅ DF = 14, ∠A = (6x + 4)°, ∠D = (8x – 12)°, BC = 3y – 2, and EF = y + 6. If ∠A > ∠D, find the integer values of x such that all angles are valid, and express the condition on y so that BC > EF is consistent with the Hinge Theorem.

$$6x + 4 > 8x - 12 \Rightarrow x < 8, \quad 3y - 2 > y + 6 \Rightarrow y > 4$$

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# The Hinge Theorem and Its Converse — Answer Key

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## Answer Key

**1. Answer: DF > AC**

- The Hinge Theorem states: if two sides of one triangle are congruent to two sides of another triangle and the included angles are not equal, then the longer third side is opposite the larger included angle.
- Since ∠E (80°) > ∠B (50°), the side opposite ∠E, which is DF, is longer than the side opposite ∠B, which is AC.

**2. Answer: PR > XZ**

- Identify the included angles between the two pairs of congruent sides: ∠Q and ∠Y.
- Since ∠Q (95°) > ∠Y (40°), by the Hinge Theorem, the third side opposite ∠Q (which is PR) is longer than the third side opposite ∠Y (which is XZ).

**3. Answer: ∠N > ∠S**

- The Converse of the Hinge Theorem states: if two sides of one triangle are congruent to two sides of another and the third sides are not equal, then the larger included angle is opposite the longer third side.
- Since MO (12) > RT (7), the included angle opposite MO, which is ∠N, is greater than the included angle opposite RT, which is ∠S.

**4. Answer: SK > YU**

- Identify the two pairs of congruent sides: SA ≅ YO and AK ≅ OU. The included angles are ∠A = 90° and ∠O = 70°.
- By the Hinge Theorem, since ∠A (90°) > ∠O (70°), the third side SK is longer than YU.

**5. Answer: ∠A > ∠D**

- Given: AB ≅ DE and AC ≅ DF (two pairs of congruent sides). The third sides are BC = 6 and EF = 5.
- Since BC (6) > EF (5), by the Converse of the Hinge Theorem, the included angle opposite BC, which is ∠A, is greater than the included angle opposite EF, which is ∠D.

**6. Answer: JL = WY (the triangles are congruent by SAS)**

- The Hinge Theorem applies only when the included angles are NOT equal. Here ∠K = ∠X = 110°.
- Since both pairs of sides are congruent AND the included angles are equal, the triangles are congruent by SAS, so JL = WY.

**7. Answer: x < 8**

- By the Hinge Theorem, since BC > EF, the included angle ∠A must be greater than ∠D: $3x + 10 > 5x – 6$.
- Solving: $10 + 6 > 5x – 3x \rightarrow 16 > 2x \rightarrow x < 8$.

**8. Answer: x < 10 (and x > 1.25 so angles stay positive)**

- By the Hinge Theorem, since PR < SU, the included angle ∠Q must be less than ∠T: $4x – 5 < 2x + 15$.
- Solving: $2x < 20 \rightarrow x < 10$. Also, ∠Q > 0° requires $4x – 5 > 0 \rightarrow x > 1.25$. So $1.25 < x < 10$.

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**9. Answer: The included angle of the second triangle is less than 120°**

- Find the third side of the first triangle using the Law of Cosines: $c^2 = 8^2 + 13^2 - 2(8)(13)\cos(120°) = 64 + 169 + 104 = 337$, so $c \approx 18.36$ cm.
- Since $15 < 18.36$, by the Converse of the Hinge Theorem, the included angle of the second triangle is less than 120°.

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**10. Answer: x < 8 (x is a positive integer: x = 1,2,3,4,5,6,7) and y > 4**

- For $\angle A > \angle D$: $6x + 4 > 8x - 12 \rightarrow 16 > 2x \rightarrow x < 8$. Since angles must be positive, $6x + 4 > 0$ is satisfied for any positive x. Valid positive integer values: x = 1, 2, 3, 4, 5, 6, 7.
- By the Hinge Theorem, $\angle A > \angle D$ implies BC > EF: $3y - 2 > y + 6 \rightarrow 2y > 8 \rightarrow y > 4$. Both conditions together confirm consistency with the Hinge Theorem.

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