Triangle Inequality Proofs & the Hinge Theorem

Geometry Worksheet · Grade 9–11

Name: _____ Date: _____

Learning Objectives

- Write two-column proofs involving triangle inequalities using given information, the reflexive property, and theorems.
- Apply the Hinge Theorem and its Converse to compare angles and sides in triangles sharing a common side.
- Construct logical proof sequences that lead from given statements to conclusions about angle or side relationships.

Problems

1. In triangle ABC, AB = 5 and BC = 8. Which side is opposite the larger angle?

$$AB = 5,\ BC = 8$$

2. State the reason that justifies writing AC \cong AC in a two-column proof when diagonal AC is shared by two triangles.

$$AC \cong AC$$

3. In quadrilateral ABCD with diagonal AC, you are given that AB \cong CD and AC \cong AC. If BC < AD, what can you conclude about angles 1 and 4 formed at vertex C?

$$BC < AD$$

4. The Hinge Theorem states that if two sides of one triangle are congruent to two sides of another triangle and the included angle of the first is larger, what is true about the third sides?

5. In two triangles sharing side PQ, with PR = XQ and PQ = PQ, if m∠RPQ = 70° and m∠XPQ = 50°, which is longer: RQ or XQ?

$$m\angle RPQ = 70^\circ,\ m\angle XPQ = 50^\circ$$

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6. Write the first two statements and reasons for a two-column proof showing m∠1 < m∠4 in quadrilateral ABCD, given AB \cong CD and BC < AD.

$$AB \cong CD,\ BC < AD$$

7. In triangle DEF, m∠D = 95°, m∠E = 50°, and m∠F = 35°. List the sides in order from shortest to longest.

$$m\angle D = 95^\circ,\ m\angle E = 50^\circ,\ m\angle F = 35^\circ$$

8. Given triangles ABC and DEF where AB = DE = 7, AC = DF = 10, and m∠A = 80°, m∠D = 65°, use the Hinge Theorem to compare BC and EF.

$$AB = DE = 7,\ AC = DF = 10,\ m\angle A = 80^\circ,\ m\angle D = 65^\circ$$

9. In quadrilateral PQRS with diagonal PR, you are given PQ \cong SR, PR \cong PR, and QR > PS. Write a complete four-step two-column proof to show m∠QPR > m∠SRP.

$$PQ \cong SR,\ QR > PS$$

10. In triangles ABD and CBD sharing side BD, with AB = CB and given that m∠ABD > m∠CBD, prove that AD > CD using a full two-column proof and identify all theorems used.

$$AB = CB,\ m\angle ABD > m\angle CBD$$

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Triangle Inequality Proofs & the Hinge Theorem — Answer Key

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Answer Key

1. Answer: BC is opposite the larger angle (angle A > angle C)

- The Triangle Inequality theorem states the larger side is opposite the larger angle.
- Since BC = 8 > AB = 5, angle A (opposite BC) > angle C (opposite AB).

2. Answer: Reflexive Property of Congruence

- Any segment is congruent to itself.
- This is the Reflexive Property of Congruence.

3. Answer: m∠1 < m∠4 (by the Converse of the Hinge Theorem)

- Triangles ABC and CDA share side AC, and AB ≅ CD (given).
- Since the third side BC < AD, by the Converse of the Hinge Theorem, the included angle opposite BC is smaller, so m∠1 < m∠4.

4. Answer: The third side opposite the larger included angle is longer.

- The Hinge Theorem (SAS Inequality) compares third sides based on included angles.
- If the included angle is larger, the side opposite it must also be larger.

5. Answer: RQ > XQ

- Both triangles share side PQ, and PR = XQ is given.
- By the Hinge Theorem, since m∠RPQ > m∠XPQ, the side opposite the larger angle is longer, so RQ > XQ.

6. Answer: Statement 1: AB ≅ CD, Reason: Given. Statement 2: AC ≅ AC, Reason: Reflexive Property.

- Always begin a proof by listing given information as Statement 1 with reason 'Given'.
- Identify shared sides or angles and use the Reflexive Property as the next statement.

7. Answer: EF < DF < DE

- The smallest side is opposite the smallest angle: side EF is opposite ∠D? No — EF is opposite ∠D (95°), DF is opposite ∠E (50°), DE is opposite ∠F (35°).
- Wait — correct mapping: side opposite ∠F (35°) = DE is shortest; side opposite ∠E (50°) = DF; side opposite ∠D (95°) = EF is longest. Order: DE < DF < EF.

8. Answer: BC > EF

- Two sides of triangle ABC equal two sides of triangle DEF (AB=DE, AC=DF).
- Since the included angle m∠A = 80° > m∠D = 65°, by the Hinge Theorem BC > EF.

9. Answer: m∠QPR > m∠SRP by the Converse of the Hinge Theorem

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- Statement 1: PQ ≅ SR, Reason: Given.
- Statement 2: PR ≅ PR, Reason: Reflexive Property.
- Statement 3: QR > PS, Reason: Given.
- Statement 4: m∠QPR > m∠SRP, Reason: Converse of the Hinge Theorem (SAS Inequality).

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10. Answer: AD > CD by the Hinge Theorem

- Statement 1: AB = CB, Reason: Given.
- Statement 2: BD = BD, Reason: Reflexive Property of Congruence.
- Statement 3: m∠ABD > m∠CBD, Reason: Given.
- Statement 4: AD > CD, Reason: Hinge Theorem — if two sides of one triangle are congruent to two sides of another and the included angle of the first is larger, then the opposite side is longer.

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