

Geometry: Similarity in Right Triangles & Geometric Mean

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Name: _____ Date: _____ Score: / 10

DIRECTIONS

Solve each problem using the altitude of a right triangle theorem and geometric mean. Show all work.

- 1 Find the geometric mean of 4 and 9:

$$\frac{4}{x} = \frac{x}{9}$$

Answer: _____

- 3 Identify all 3 similar triangles:

$\triangle XYZ$ with altitude YW to hypotenuse XZ

Answer: _____

- 5 Find the altitude h given the segments:

Hypotenuse segments: 4 and 9, $\frac{4}{h} = \frac{h}{9}$

Answer: _____

- 7 Find the altitude h given the segments:

Hypotenuse segments: 6 and 10, $h = \sqrt{6 \cdot 10}$

Answer: _____

- 9 Find both legs given the segments:

Segments on hypotenuse: $p = 3$, $q = 12$; find legs a and b

Answer: _____

- 2 Find the geometric mean of 5 and 20:

$$\frac{5}{x} = \frac{x}{20}$$

Answer: _____

- 4 Find the geometric mean of 3 and 27:

$$\frac{3}{x} = \frac{x}{27}$$

Answer: _____

- 6 Find leg a using the corollary:

Whole hypotenuse = 13, near segment = 4, $\frac{4}{a} = \frac{a}{13}$

Answer: _____

- 8 Find x using triangle similarity:

$\triangle ABC \sim \triangle ADB$, $\frac{AD}{AB} = \frac{AB}{AC}$, $AD = 5$, $AC = 20$

Answer: _____

- 10 Find x , y , and h in the right triangle:

$\triangle ABC$, altitude to hypotenuse, $AD = 8$, $DC = 2$, find h , AB , BC

Answer: _____



Answer Key & Solutions

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Emphasize that when an altitude is drawn to the hypotenuse of a right triangle, it creates two smaller triangles that are each similar to the original and to each other (AA Similarity). Remind students that the geometric mean relationship comes directly from setting up proportions between corresponding sides of similar triangles.

TEACHER NOTE

1 Find the geometric mean of 4 and 9:

$$\frac{4}{x} = \frac{x}{9}$$

$$= x = 6$$

Cross-multiply to get $x^2 = 36$, then take the square root to get $x = 6$.

2 Identify all 3 similar triangles:

$\triangle XYZ$ with altitude YW to hypotenuse XZ

$$= \triangle XYZ \sim \triangle XWY \sim \triangle YWZ$$

By the Altitude of a Right Triangle Theorem, the altitude to the hypotenuse creates two sub-triangles each similar to the original and to each other.

5 Find the altitude h given the segments:

Hypotenuse segments: 4 and 9, $\frac{4}{h} = \frac{h}{9}$

$$= h = 6$$

By the geometric mean (altitude) corollary, $h^2 = 4 \cdot 9 = 36$, so $h = 6$.

7 Find the altitude h given the segments:

Hypotenuse segments: 6 and 10, $h = \sqrt{6 \cdot 10}$

$$= h = \sqrt{60} \approx 7.75$$

Using the altitude corollary, $h^2 = 6 \cdot 10 = 60$, so $h = \sqrt{60} = 2\sqrt{15} \approx 7.75$.

9 Find both legs given the segments:

Segments on hypotenuse: $p = 3$, $q = 12$; find legs a and b

$$= a = 6, b = \sqrt{180} = 6\sqrt{5} \approx 13.42$$

Using the leg corollary: $a^2 = p(p+q) = 3 \cdot 15 = 45$ so $a = 3\sqrt{5} \approx 6.71$, and $b^2 = q(p+q) = 12 \cdot 15 = 180$ so $b = 6\sqrt{5} \approx 13.42$.

2 Find the geometric mean of 5 and 20:

$$\frac{5}{x} = \frac{x}{20}$$

$$= x = 10$$

Cross-multiply to get $x^2 = 100$, then take the square root to get $x = 10$.

4 Find the geometric mean of 3 and 27:

$$\frac{3}{x} = \frac{x}{27}$$

$$= x = 9$$

Cross-multiply to get $x^2 = 81$, then take the square root to get $x = 9$.

6 Find leg a using the corollary:

Whole hypotenuse = 13, near segment = 4, $\frac{4}{a} = \frac{a}{13}$

$$= a = \sqrt{52} \approx 7.21$$

By the geometric mean (leg) corollary, $a^2 = 4 \cdot 13 = 52$, so $a = \sqrt{52} = 2\sqrt{13} \approx 7.21$.

8 Find x using triangle similarity:

$\triangle ABC \sim \triangle ADB$, $\frac{AD}{AB} = \frac{AB}{AC}$, $AD = 5$, $AC = 20$

$$= AB = 10$$

Set up $AB^2 = AD \cdot AC = 5 \cdot 20 = 100$, so $AB = 10$.

10 Find x , y , and h in the right triangle:

$\triangle ABC$, altitude to hypotenuse, $AD = 8$, $DC = 2$, find h , AB , BC

$$= h = 4, AB = 4\sqrt{5}, BC = 2\sqrt{5}$$

Hypotenuse $AC = 8 + 2 = 10$. Altitude $h^2 = 8 \cdot 2 = 16$, $h = 4$. Leg $AB^2 = 8 \cdot 10 = 80$, $AB = 4\sqrt{5} \approx 8.94$. Leg $BC^2 = 2 \cdot 10 = 20$, $BC = 2\sqrt{5} \approx 4.47$.

