Chords, Arcs, and Central Angles in Circles

Geometry Worksheet · Grade 8–10

Name: ________________________________ Date: ___________________

Learning Objectives

- Identify and distinguish chords, secant lines, and tangent lines in a circle
- Apply the theorem that congruent central angles have congruent arcs and congruent chords
- Solve for unknown arc measures and chord lengths using circle theorems

Problems

1. A line segment has both endpoints on a circle. What is this segment called?

2. True or False: A tangent line intersects a circle at exactly two points.

3. In circle O, chord PQ is part of line PQ extended beyond the circle. What is the extended line called?

4. In circle O, central angle AOB = 75°. What is the measure of arc AB?

$$\angle AOB = 75^\circ$$

5. In circle O, arc AB ≅ arc CD. What can you conclude about central angles AOB and COD?

$$\overset{\frown}{AB} \cong \overset{\frown}{CD}$$

6. In circle O, central angle AOB = 110° and central angle COD = 110°. What is the relationship between chord AB and chord CD?

$$\angle AOB = \angle COD = 110^\circ$$

7. In circle O with radius 10 cm, central angle AOB = 60°. If central angle COD is also 60°, find the length of chord CD given that chord AB = 10 cm.

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$$\angle AOB = \angle COD = 60^\circ,\ AB = 10\text{ cm}$$

8. In circle O, arc AB = (3x + 10)° and arc CD = (5x – 14)°, and it is given that ∠AOB ≅ ∠COD. Find x and the measure of each arc.

$$3x + 10 = 5x - 14$$

9. In circle O, chord AB = (4x + 3) cm and chord CD = (6x – 9) cm with ∠AOB ≅ ∠COD. Find x and the length of each chord.

$$4x + 3 = 6x - 9$$

10. In circle O with radius r, two chords AB and CD satisfy AB ≅ CD. Central angle AOB = (7x – 4)° and central angle COD = (4x + 23)°. Find x, the measure of each central angle, and state the measure of each associated arc.

$$7x - 4 = 4x + 23$$

Chords, Arcs, and Central Angles in Circles — Answer Key

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Answer Key

1. Answer: Chord

- Recall the definition: a chord is a segment whose endpoints lie on the circle.
- Therefore, the segment is called a chord.

2. Answer: False

- A tangent line touches the circle at exactly one point (the point of tangency).
- A secant line intersects the circle at two points, not a tangent. The answer is False.

3. Answer: Secant line

- When a chord is extended into a full line that cuts through the circle at two points, it becomes a secant line.
- Therefore, the extended line is called a secant line.

4. Answer: 75°

- By the Central Angle-Arc Theorem, the measure of an arc equals the measure of its central angle.
- Therefore, arc $AB = 75°$.

5. Answer: $\angle AOB \cong \angle COD$

- By the converse of the Congruent Arcs Theorem, if two arcs in the same circle are congruent, their central angles are congruent.
- Therefore, $\angle AOB \cong \angle COD$.

6. Answer: $AB \cong CD$ (chords are congruent)

- By the Congruent Central Angles Theorem, congruent central angles in the same circle have congruent chords.
- Since $\angle AOB = \angle COD$, it follows that chord $AB \cong$ chord CD.

7. Answer: $CD = 10$ cm

- Congruent central angles in the same circle produce congruent chords.
- Since $\angle AOB \cong \angle COD$, chord $AB \cong$ chord CD, so $CD = 10$ cm.

8. Answer: $x = 12$, each arc = 46°

- Since congruent central angles produce congruent arcs, set arc AB = arc CD: $3x + 10 = 5x - 14$.
- Solve: $10 + 14 = 5x - 3x \rightarrow 24 = 2x \rightarrow x = 12$.
- Substitute: arc $AB = 3(12) + 10 = 46°$, and arc $CD = 5(12) - 14 = 46°$. ✓

9. Answer: $x = 6$, each chord = 27 cm

- By the Congruent Central Angles Theorem, congruent central angles give congruent chords: $4x + 3 = 6x - 9$.
- Solve: $3 + 9 = 6x - 4x \rightarrow 12 = 2x \rightarrow x = 6$.

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- Substitute: AB = 4(6) + 3 = 27 cm, CD = 6(6) – 9 = 27 cm. ✓

10. Answer: x = 9, each central angle = 59°, each arc = 59°

- Congruent chords in the same circle have congruent central angles, so set $\angle AOB = \angle COD$: $7x - 4 = 4x + 23$.
- Solve: $7x - 4x = 23 + 4 \rightarrow 3x = 27 \rightarrow x = 9$.
- Substitute: $\angle AOB = 7(9) - 4 = 59°$ and $\angle COD = 4(9) + 23 = 59°$.
- By the Central Angle-Arc Theorem, arc AB = 59° and arc CD = 59°.

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