Solving Oblique Triangles Using the Law of Cosines

Trigonometry Worksheet · Grade 10–12

Name: ______________________________ Date: __________________

Learning Objectives

- Apply the Law of Cosines to find missing sides in SAS (Side-Angle-Side) oblique triangles
- Apply the Law of Cosines to find missing angles in SSS (Side-Side-Side) oblique triangles
- Use the alternative form of the Law of Cosines and inverse cosine to determine angle measures including obtuse angles

Problems

1. In triangle ABC, a = 5, b = 7, and c = 8. Use the alternative form of the Law of Cosines to find angle C.

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

2. In triangle ABC, a = 6, b = 8, and c = 10. Use the Law of Cosines to determine whether angle C is acute, right, or obtuse.

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

3. In triangle ABC, sides a = 8, b = 19, and c = 14. Identify the longest side and find the angle opposite to it.

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

4. In triangle ABC, a = 10, c = 12, and the included angle B = 60°. Use the standard form of the Law of Cosines to find side b.

$$b^2 = a^2 + c^2 - 2ac\cos B$$

5. In triangle ABC, a = 7, b = 9, and c = 11. Find all three angles of the triangle using the Law of Cosines.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

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6. In triangle ABC, a = 5, b = 6, and angle C = 120°. Find the length of side c.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

7. In triangle ABC, a = 15, b = 20, and c = 25. Find angle A using the alternative form of the Law of Cosines.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

8. Two sides of a triangle measure 13 cm and 17 cm, and the angle between them is 75°. Find the length of the third side and the measure of the largest remaining angle.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

9. A triangular plot of land has sides measuring 120 m, 150 m, and 200 m. Find the largest interior angle of the plot.

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

10. In triangle ABC, a = 9, b = 12, and c = 16. First find the obtuse angle using the Law of Cosines, then use the Law of Sines to find a second angle, and finally determine the third angle by subtraction.

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \quad \text{then} \quad \frac{\sin A}{a} = \frac{\sin C}{c}$$

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Solving Oblique Triangles Using the Law of Cosines — Answer Key

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Answer Key

1. Answer: C ≈ 75.5°

- Substitute: cos C = (25 + 49 – 64) / (2 · 5 · 7) = 10 / 70 ≈ 0.1429
- Apply inverse cosine: C = $\cos^{-1}$(0.1429) ≈ 75.5°

2. Answer: C = 90° (right angle)

- Substitute: cos C = (36 + 64 – 100) / (2 · 6 · 8) = 0 / 96 = 0
- Apply inverse cosine: C = $\cos^{-1}$(0) = 90°, confirming a right angle

3. Answer: B ≈ 116.8°

- Longest side is b = 19; substitute: cos B = (64 + 196 – 361) / (2 · 8 · 14) = –101 / 224 ≈ –0.4509
- Apply inverse cosine: B = $\cos^{-1}$(–0.4509) ≈ 116.8°

4. Answer: b ≈ 11.1

- Substitute: b^2 = 100 + 144 – 2(10)(12)cos(60°) = 244 – 240(0.5) = 244 – 120 = 124
- Take square root: b = √124 ≈ 11.1

5. Answer: A ≈ 38.2°, B ≈ 52.6°, C ≈ 89.2°

- Find A: cos A = (81 + 121 – 49)/(2·9·11) = 153/198 ≈ 0.7727 → A ≈ 39.5°; find B similarly using cos B = (49+121–81)/(2·7·11) ≈ 0.6039 → B ≈ 52.8°
- Find C: C = 180° – A – B ≈ 180° – 39.5° – 52.8° = 87.7°; verify with law of cosines directly

6. Answer: c ≈ 9.54

- Substitute: c^2 = 25 + 36 – 2(5)(6)cos(120°) = 61 – 60(–0.5) = 61 + 30 = 91
- Take square root: c = √91 ≈ 9.54

7. Answer: A = 36.9°

- Substitute: cos A = (400 + 625 – 225) / (2 · 20 · 25) = 800 / 1000 = 0.8
- Apply inverse cosine: A = $\cos^{-1}$(0.8) ≈ 36.9°

8. Answer: c ≈ 18.0 cm; largest remaining angle ≈ 70.3°

- Find c: c^2 = 169 + 289 – 2(13)(17)cos(75°) = 458 – 442(0.2588) ≈ 458 – 114.4 = 343.6 → c ≈ 18.0 cm
- Use Law of Cosines to find the angle opposite b=17: cos B = (169 + 324 – 289)/(2·13·18) = 204/468 ≈ 0.4359 → B ≈ 64.2°; angle opposite a=13 is 180°–75°–64.2° = 40.8°; largest remaining angle ≈ 64.2°

9. Answer: C ≈ 93.8°

- The largest angle is opposite the longest side c = 200 m; substitute: cos C = (14400 + 22500 – 40000) / (2·120·150) = –3100 / 36000 ≈ –0.0861

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- Apply inverse cosine: $C = \cos^{-1}(-0.0861) \approx 94.9°$

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10. Answer: $C \approx 104.5°$, $A \approx 32.1°$, $B \approx 43.4°$

- Find C (obtuse, opposite longest side c=16): $\cos C = (81 + 144 - 256)/(2·9·12) = -31/216 \approx -0.1435 \to C = \cos^{-1}(-0.1435) \approx 104.5°$
- Use Law of Sines: $\sin A / 9 = \sin(104.5°) / 16 \to \sin A = 9·\sin(104.5°)/16 \approx 0.5432 \to A \approx 32.9°$; then $B = 180° - 104.5° - 32.9° \approx 42.6°$

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