Angles of a Polygon Using Diagonals

Geometry Worksheet · Grade 7–10

Name: _____ Date: _____

Learning Objectives

- Calculate the number of diagonals in a polygon using the diagonal formula
- Determine the sum of interior angles of a polygon using triangles formed by diagonals
- Find individual interior and exterior angle measures of regular polygons

Problems

1. How many diagonals can be drawn from a single vertex of a regular quadrilateral (square)?

2. How many triangles are formed when all diagonals are drawn from one vertex of a regular quadrilateral (4 sides)?

3. What is the sum of interior angles of a regular quadrilateral using the triangles formed by its diagonals?

$$S = (n - 2) \times 180^\circ$$

4. Find the measure of each interior angle of a regular quadrilateral (square).

$$\text{Each angle} = \frac{S}{n}$$

5. Using the diagonal method, find the sum of interior angles of a regular pentagon (5 sides).

$$S = (n - 2) \times 180^\circ$$

6. Find each interior angle of a regular pentagon using the diagonals method.

$$\text{Each angle} = \frac{(n - 2) \times 180^\circ}{n}$$

7. Find the total number of diagonals in a regular hexagon using the diagonals formula.

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$$D = \frac{n(n-3)}{2}$$

8. A regular polygon has an interior angle sum of 1080 degrees. How many sides does it have, and what is each interior angle?

$$n = \frac{S}{180^\circ} + 2$$

9. A regular polygon has 20 diagonals. Determine the number of sides of the polygon, then find the sum of its interior angles and the measure of each interior angle.

$$D = \frac{n(n-3)}{2}$$

10. A regular polygon has 35 diagonals. Find the number of sides, the sum of interior angles, each interior angle, and each exterior angle.

$$D = \frac{n(n-3)}{2}, \quad \text{Exterior angle} = \frac{360^\circ}{n}$$

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Angles of a Polygon Using Diagonals — Answer Key

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Answer Key

1. Answer: 1 diagonal

- A diagonal connects two non-adjacent vertices.
- A square has 4 vertices. From one vertex, you can connect to 4 - 1 - 2 = 1 non-adjacent vertex.
- So exactly 1 diagonal can be drawn from a single vertex.

2. Answer: 2 triangles

- When diagonals are drawn from one vertex of an n-sided polygon, the number of triangles formed is n - 2.
- For a quadrilateral: 4 - 2 = 2 triangles.

3. Answer: 360°

- Drawing a diagonal from one vertex divides the quadrilateral into 2 triangles.
- Each triangle has an angle sum of 180°.
- Total interior angle sum = 2 × 180° = 360°.

4. Answer: 90°

- The sum of interior angles of a quadrilateral is 360°.
- A regular quadrilateral has 4 equal angles.
- Each angle = 360° ÷ 4 = 90°.

5. Answer: 540°

- Draw diagonals from one vertex of the pentagon — this creates 5 - 2 = 3 triangles.
- Sum of interior angles = 3 × 180° = 540°.

6. Answer: 108°

- Sum of interior angles of a pentagon = (5 - 2) × 180° = 540°.
- A regular pentagon has 5 equal angles.
- Each interior angle = 540° ÷ 5 = 108°.

7. Answer: 9 diagonals

- Use the formula D = n(n - 3) / 2, where n = 6.
- D = 6(6 - 3) / 2 = 6 × 3 / 2 = 18 / 2 = 9.
- A regular hexagon has 9 diagonals.

8. Answer: 8 sides; each interior angle = 135°

- Use S = (n - 2) × 180°, so n - 2 = S / 180° = 1080° / 180° = 6.
- Therefore n = 6 + 2 = 8 sides (regular octagon).
- Each interior angle = 1080° ÷ 8 = 135°.

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9. Answer: 8 sides; interior angle sum = 1080°; each angle = 135°

- Set D = n(n - 3)/2 = 20, so n(n - 3) = 40.
- Try n = 8: 8 × (8 - 3) = 8 × 5 = 40. ✓ So the polygon has 8 sides.
- Interior angle sum = (8 - 2) × 180° = 6 × 180° = 1080°.
- Each interior angle = 1080° ÷ 8 = 135°.

10. Answer: 10 sides; interior sum = 1440°; each interior angle = 144°; each exterior angle = 36°

- Set D = n(n - 3)/2 = 35, so n(n - 3) = 70.
- Try n = 10: 10 × (10 - 3) = 10 × 7 = 70. ✓ So the polygon has 10 sides (decagon).
- Interior angle sum = (10 - 2) × 180° = 8 × 180° = 1440°.
- Each interior angle = 1440° ÷ 10 = 144°.
- Each exterior angle = 360° ÷ 10 = 36°. Check: 144° + 36° = 180°. ✓

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