Chords, Arcs, and Central Angles in Circles

Geometry Worksheet · Grade 9–11

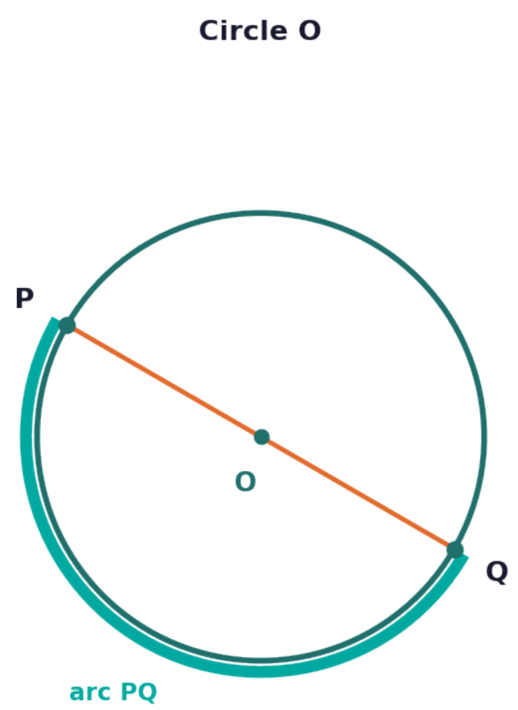
Name: ________________________________ Date: __________________

Learning Objectives

- Identify and distinguish chords, secants, and tangent lines in a circle
- Apply the theorem that congruent central angles have congruent arcs and congruent chords
- Use chord and arc relationships to solve for unknown angle measures, arc measures, and chord lengths

Problems

1. In circle O, segment PQ has both of its endpoints on the circle. What is the name of segment PQ, and what arc does it produce?



2. Explain the difference between a chord, a secant line, and a tangent line with respect to a circle. How many times does each line intersect the circle?

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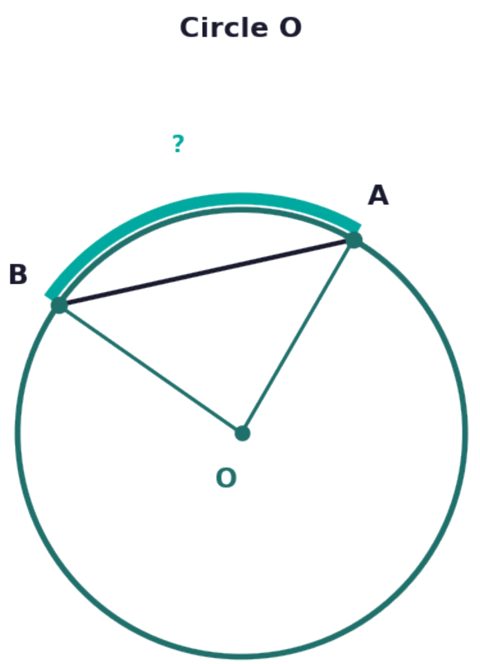
3. In circle O, the central angle AOB measures 85 degrees. What is the measure of arc AB?



4. In circle O, central angle AOB is congruent to central angle COD. If arc AB measures 112 degrees, what is the measure of arc CD?

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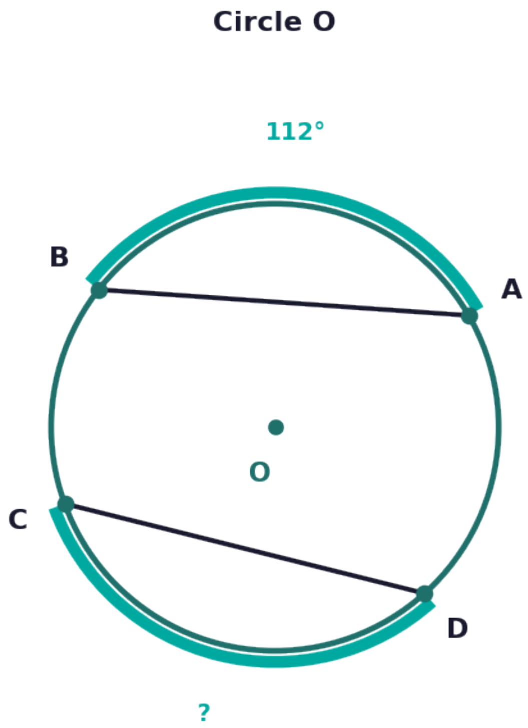

5. In circle O, arc AB measures 98 degrees and arc CD measures 98 degrees. Are central angles AOB and COD congruent? State the theorem that justifies your answer.

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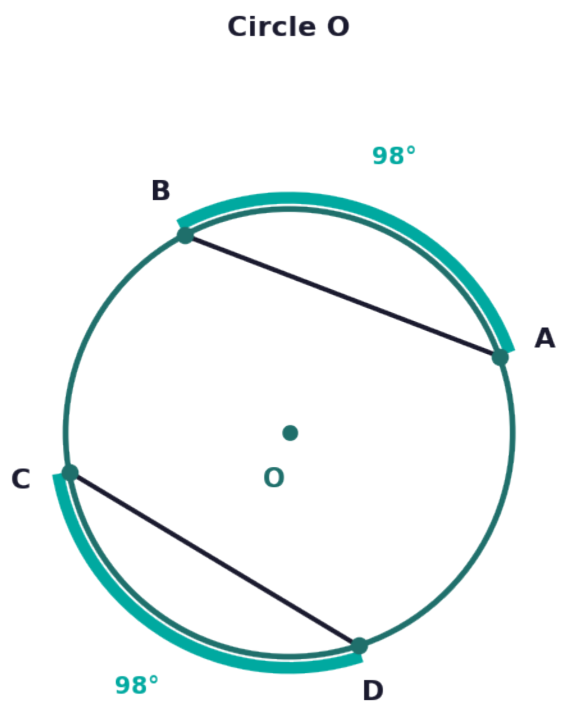
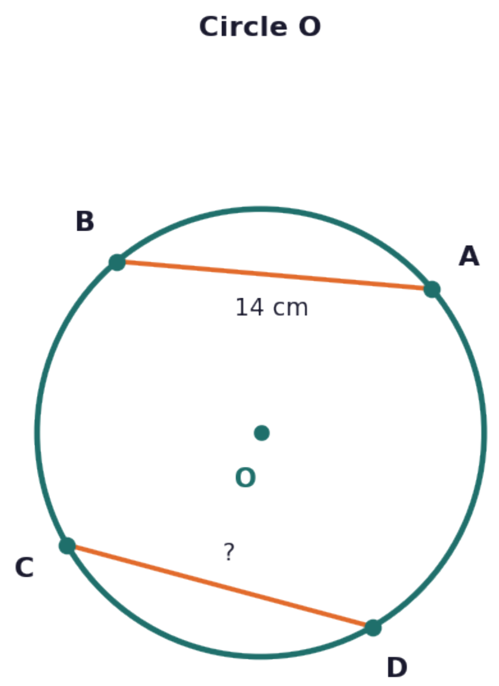


6. In circle O, central angle AOB is congruent to central angle COD. If chord AB has length 14 cm, what is the length of chord CD? Name the theorem used.



7. In circle O with radius 13, a perpendicular from the center O to chord AB meets the chord at point M. If OM equals 5, find the length of chord AB using the Pythagorean Theorem.

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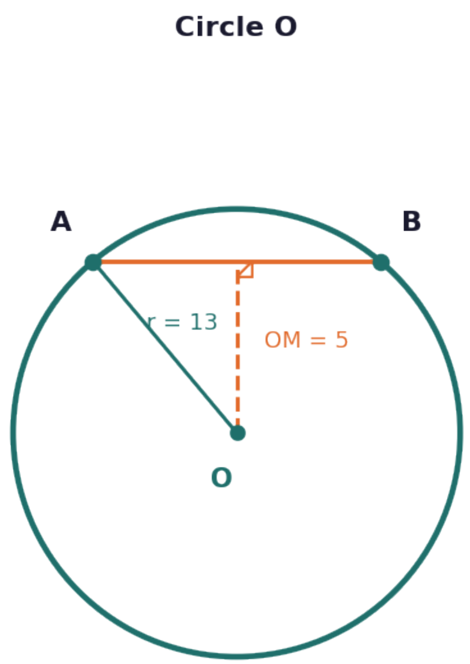

8. Two chords AB and CD in circle O are equidistant from the center (each perpendicular distance from O equals 6). The radius of the circle is 10. Prove that the two chords must be congruent by finding each chord's length.

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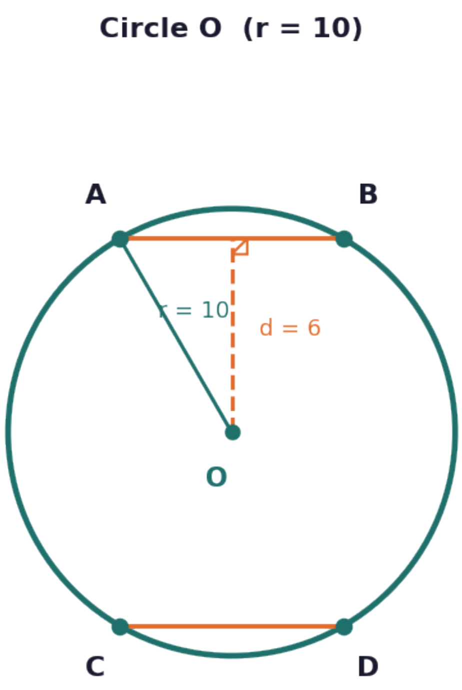

9. In circle O, arc AB measures (3x + 10) degrees and arc CD measures (5x minus 14) degrees. Central angle AOB is congruent to central angle COD. Find x and then find the measure of each arc.

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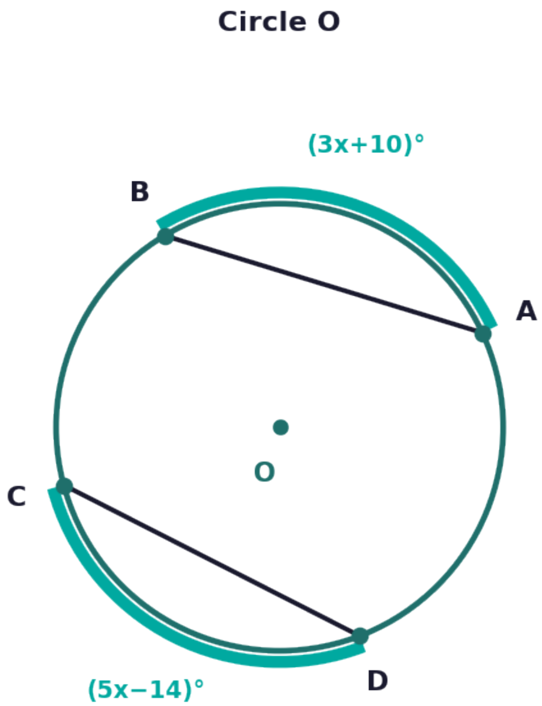

10. In two congruent circles O1 and O2 each with radius 17, chord AB in circle O1 has length 30 and is associated with central angle AO1B. Chord CD in circle O2 is associated with central angle CO2D. If arc AB equals arc CD, find: (a) the distance from center O1 to chord AB, (b) the length of chord CD, and (c) the distance from center O2 to chord CD.

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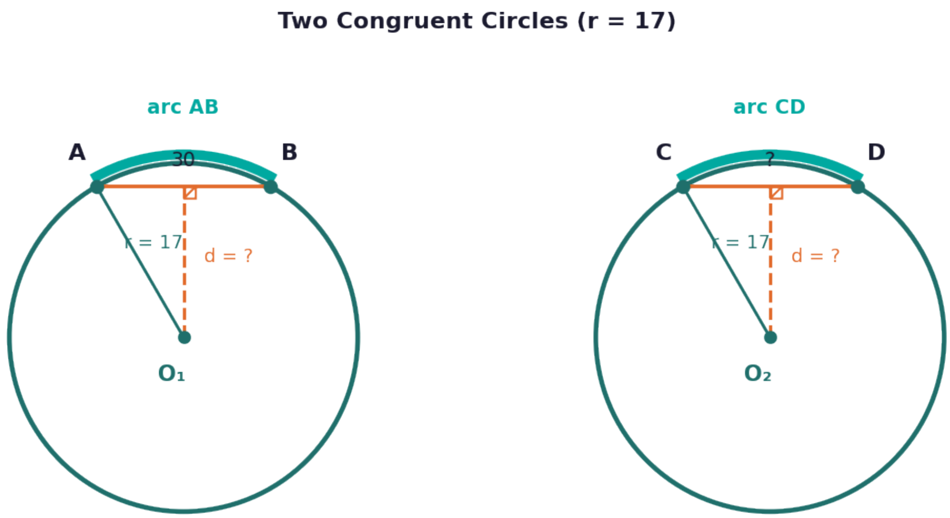

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Chords, Arcs, and Central Angles in Circles — Answer Key

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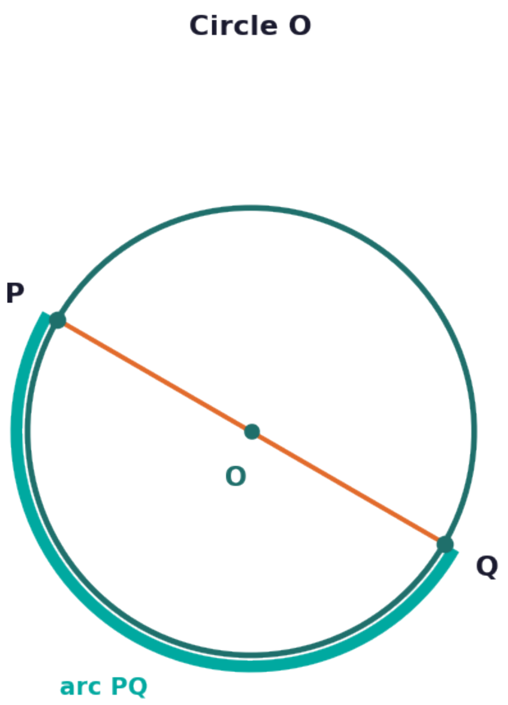
Answer Key

1. Answer: PQ is a chord; it produces arc PQ

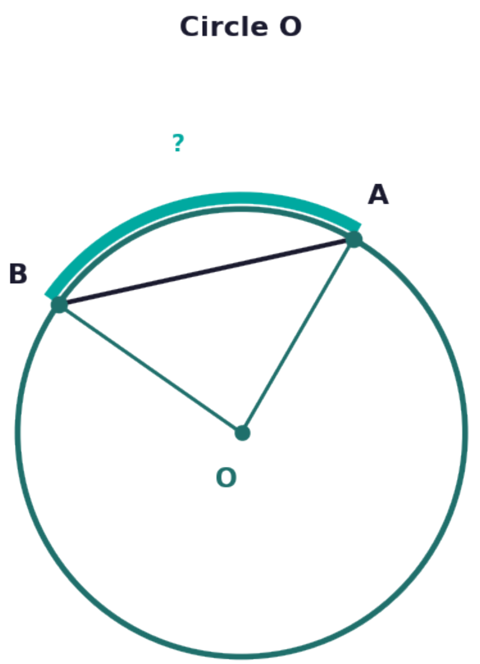


- A chord is a line segment whose both endpoints lie on the circle.
- Segment PQ has endpoints P and Q on circle O, so it is a chord.
- Every chord is associated with an arc — the arc cut off by its endpoints, called arc PQ.

2. Answer: Chord: segment with 2 endpoints ON the circle; Secant: full line crossing the circle at 2 points; Tangent: line touching the circle at exactly 1 point

- A chord is a segment that lies inside the circle with both endpoints ON the circle — it intersects the circle at 2 points.
- A secant is a line that extends through the circle, intersecting it at exactly 2 different points (the chord is the part of the secant inside the circle).
- A tangent is a line that touches the circle at exactly 1 point (the point of tangency) and does not cross it.

3. Answer: arc AB = 85°



- A central angle has its vertex at the center of the circle.
- By definition, the measure of a central angle equals the measure of its intercepted arc.
- Since central angle AOB = 85°, arc AB = 85°.

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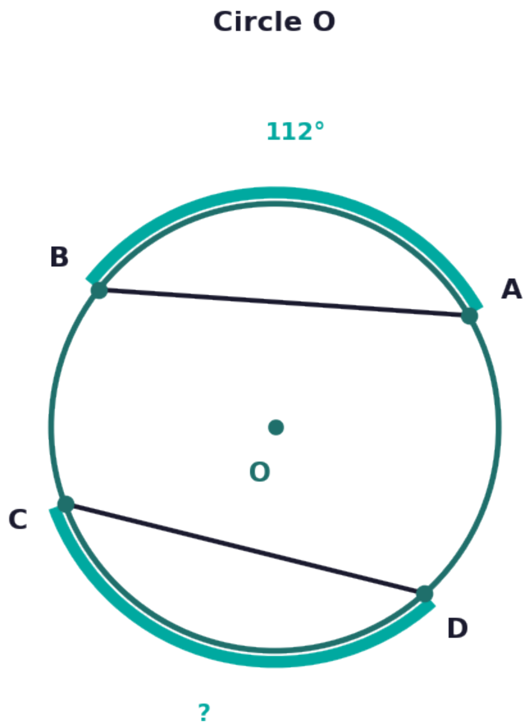
4. Answer: arc CD = 112°



- Theorem: Within a circle, congruent central angles have congruent arcs.
- Since angle AOB ≅ angle COD, it follows that arc AB ≅ arc CD.
- arc AB = 112°, therefore arc CD = 112°.

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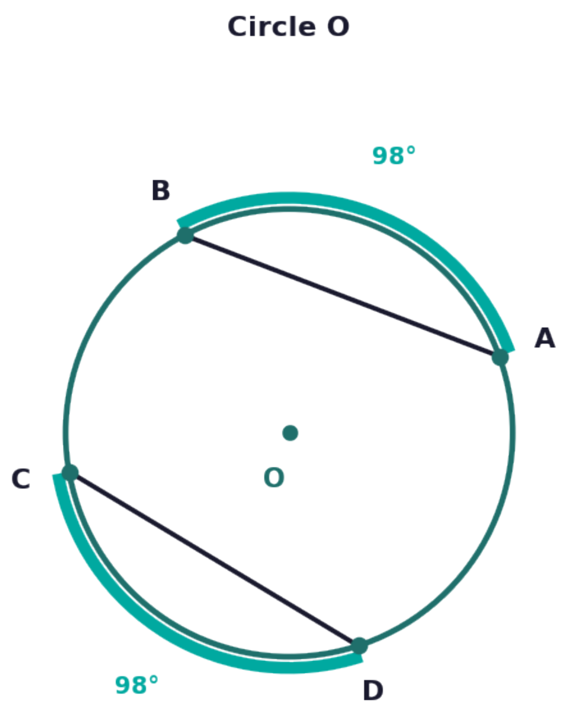
5. Answer: Yes, angle AOB ≅ angle COD, by the Converse: congruent arcs have congruent central angles



- The converse theorem states: Within a circle, congruent arcs have congruent central angles.
- arc AB = 98° and arc CD = 98°, so arc AB ≅ arc CD.
- Therefore central angle AOB ≅ central angle COD, each measuring 98°.

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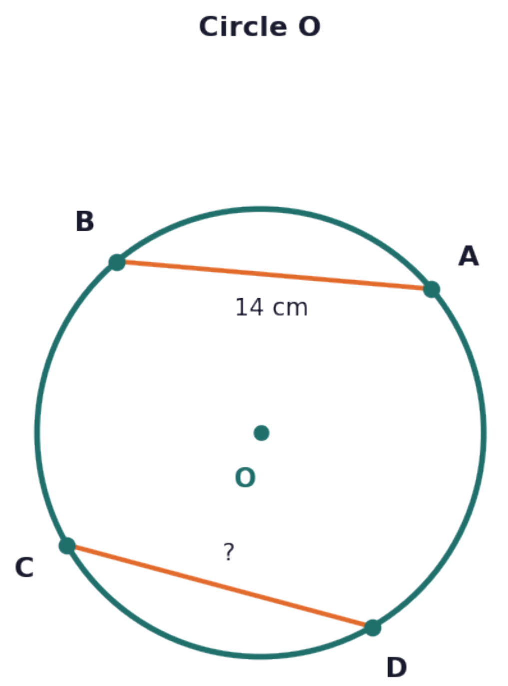
6. Answer: chord CD = 14 cm; Theorem: congruent central angles have congruent chords



- Theorem: Within a circle, congruent central angles have congruent chords.
- Since angle AOB ≅ angle COD, chord AB ≅ chord CD.
- chord AB = 14 cm, therefore chord CD = 14 cm.

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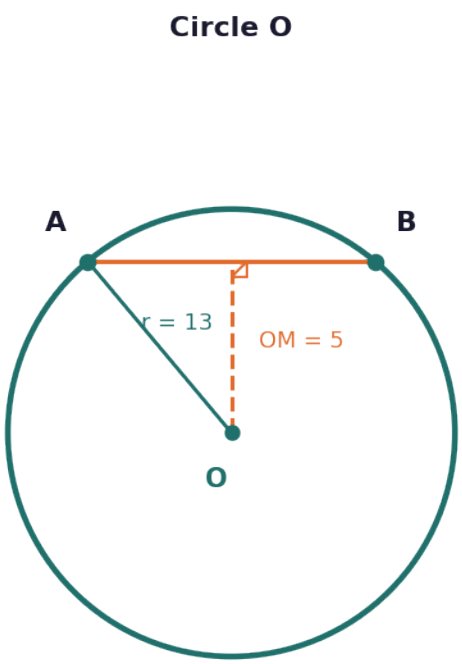
7. Answer: chord AB = 24



- The perpendicular from the center bisects the chord, so AM = MB.
- Use the Pythagorean Theorem in triangle OMA: $OA^2 = OM^2 + AM^2$.
- $13^2 = 5^2 + AM^2 \rightarrow 169 = 25 + AM^2 \rightarrow AM^2 = 144 \rightarrow AM = 12$.
- chord AB = 2 × AM = 2 × 12 = 24.

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8. Answer: chord AB = chord CD = 16

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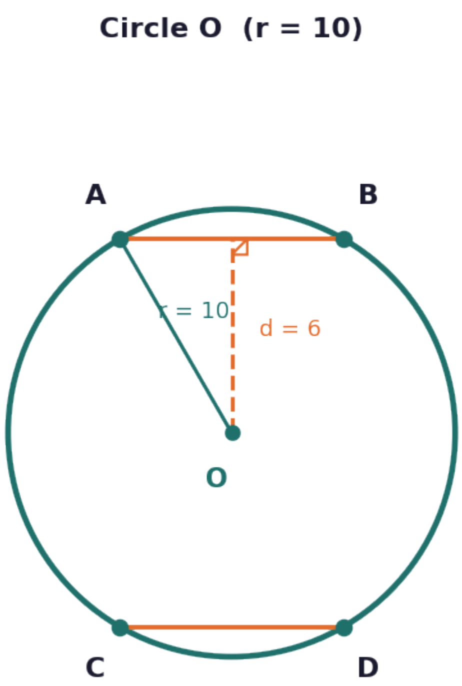

- For chord AB: $10^2 = 6^2 + (AB/2)^2 \rightarrow 100 = 36 + (AB/2)^2 \rightarrow (AB/2)^2 = 64 \rightarrow AB/2 = 8 \rightarrow AB = 16$.
- For chord CD: same calculation since $d = 6$ and $r = 10 \rightarrow CD = 16$.
- Because both chords are equidistant from the center ($d = 6$), $AB = CD = 16$. This confirms the theorem: chords equidistant from the center are congruent.

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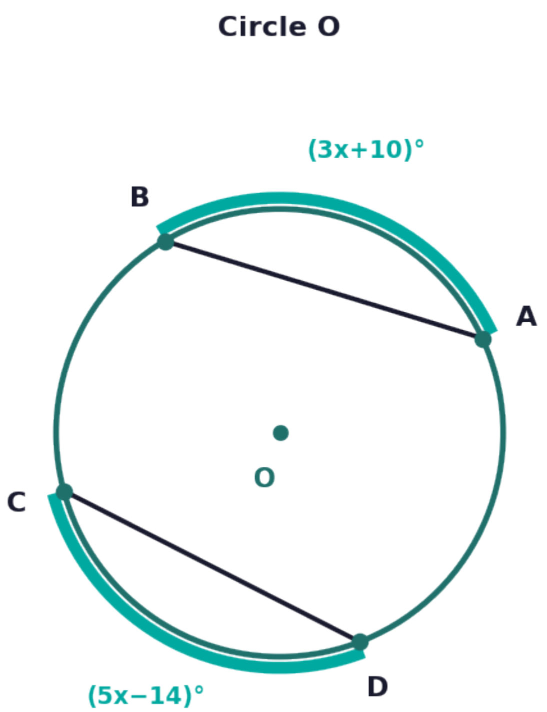
9. Answer: x = 12; arc AB = arc CD = 46°



- Congruent central angles → congruent arcs, so arc AB = arc CD.
- Set the expressions equal: 3x + 10 = 5x – 14.
- Subtract 3x from both sides: 10 = 2x – 14.
- Add 14: 24 = 2x → x = 12.
- arc AB = 3(12) + 10 = 36 + 10 = 46°; arc CD = 5(12) – 14 = 60 – 14 = 46°. ✓

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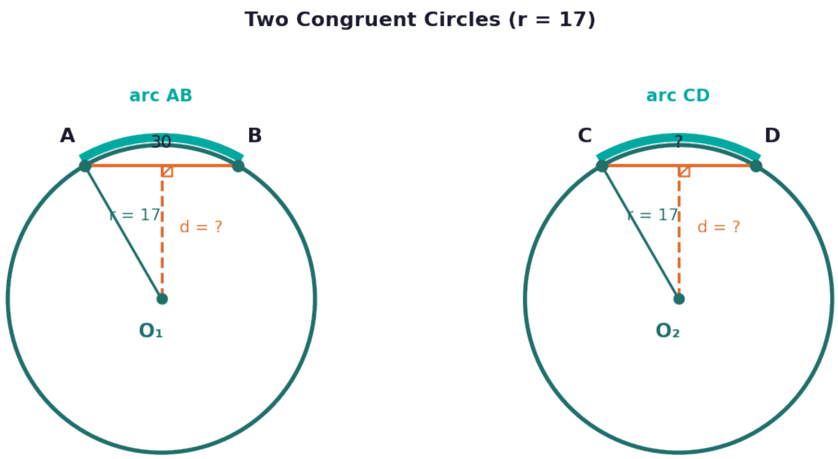
10. Answer: (a) d = 8; (b) chord CD = 30; (c) d = 8



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- (a) Distance from O1 to chord AB: $d = \sqrt{r^2 - (AB/2)^2} = \sqrt{17^2 - 15^2} = \sqrt{289 - 225} = \sqrt{64} = 8$.
- (b) Since arc AB = arc CD in congruent circles, the central angles are congruent, and therefore chord AB ≅ chord CD. So chord CD = 30.
- (c) Since chord CD = chord AB = 30 and $r = 17$ for both circles, the distance from O2 to chord CD is also $d = \sqrt{17^2 - 15^2} = 8$.
- This confirms the theorem: in congruent circles, congruent arcs correspond to congruent chords, and equidistant chords from the center are congruent.