Theorems on Chords and Arcs in Congruent Circles

Geometry Worksheet · Grade 9–11

Name: ______________________________ Date: __________________

Learning Objectives

- Apply theorems relating congruent chords to congruent arcs in a circle or in congruent circles.
- Use properties of perpendicular bisectors and central angles to find unknown measures of chords and arcs.
- Solve multi-step problems involving chord length, arc measure, and distances from the center of a circle.

Problems

1. In circle O, chord AB and chord CD are congruent. If arc AB measures 74 degrees, what is the measure of arc CD?

$$\overset{\frown}{AB} = 74^\circ, \quad AB \cong CD$$

2. Two congruent circles each have a radius of 10. In the first circle, a chord has an arc measure of 120 degrees. What is the arc measure of a congruent chord in the second circle?

$$r = 10, \quad \overset{\frown}{AB} = 120^\circ$$

3. In circle P, the diameter is perpendicular to chord MN. The diameter bisects chord MN, and half of MN measures 6 units. What is the full length of chord MN?

$$\frac{MN}{2} = 6$$

4. In circle O with radius 13, a chord is 10 units from the center. Find the length of the chord.

$$r = 13, \quad d = 10$$

5. In circle O, chords AB and CD are equidistant from the center. AB = 3x – 4 and CD = x + 10. Find the value of x and the length of each chord.

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$$AB = 3x - 4, \quad CD = x + 10$$

6. In circle O, arc RS = 5y + 10 degrees and arc TV = 8y – 14 degrees. Chords RS and TV are congruent. Find y and the measure of each arc.

$$\overset{\frown}{RS} = 5y + 10, \quad \overset{\frown}{TV} = 8y - 14$$

7. In circle O with radius 17, a chord is 8 units from the center. Another chord in the same circle is also 8 units from the center. What is the length of each chord?

$$r = 17, \quad d = 8$$

8. In two congruent circles with radius 15, chord AB in the first circle subtends a central angle of 2x + 20 degrees, and congruent chord CD in the second circle subtends a central angle of 4x – 16 degrees. Find x and the measure of each central angle.

$$\angle AOB = 2x + 20, \quad \angle COD = 4x - 16$$

9. In circle O, chord PQ is 24 units long and is 5 units from the center. A second chord RS has an arc equal to arc PQ. Find the distance of chord RS from the center and the radius of the circle.

$$PQ = 24, \quad d_{PQ} = 5$$

10. In two congruent circles with radius 25, chord AB in the first circle is 14 units from the center. In the second circle, a chord CD has an arc congruent to arc AB. Find the length of CD and the measure of central angle COD given that arc AB measures 112 degrees.

$$r = 25, \quad d_{AB} = 14, \quad \overset{\frown}{AB} = 112^\circ$$

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Theorems on Chords and Arcs in Congruent Circles — Answer Key

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Answer Key

1. Answer: arc CD = 74°

- Theorem: In the same circle, congruent chords have congruent arcs.
- Since AB ≅ CD, arc AB ≅ arc CD.
- Therefore arc CD = 74°.

2. Answer: 120°

- Theorem: In congruent circles, congruent chords have congruent arcs.
- Since the circles are congruent (equal radii) and the chords are congruent, their arcs are equal.
- The arc measure of the congruent chord in the second circle is 120°.

3. Answer: MN = 12 units

- Theorem: A diameter (or radius) perpendicular to a chord bisects the chord.
- If half of MN = 6, then MN = 2 × 6 = 12 units.

4. Answer: Chord length = $2\sqrt{69} \approx 16.61$ units

- Draw a perpendicular from center O to the chord; it bisects the chord.
- Using the Pythagorean theorem: $\text{half-chord}^2 + \text{distance}^2 = \text{radius}^2$
- $\text{half-chord}^2 = 13^2 - 10^2 = 169 - 100 = 69$
- half-chord = $\sqrt{69}$
- Full chord length = $2\sqrt{69} \approx 16.61$ units.

5. Answer: x = 7, AB = CD = 17 units

- Theorem: In the same circle, chords equidistant from the center are congruent.
- Set AB = CD: $3x - 4 = x + 10$
- $2x = 14 \rightarrow x = 7$
- AB = 3(7) – 4 = 17, CD = 7 + 10 = 17. ✓

6. Answer: y = 8, each arc = 50°

- Theorem: In the same circle, congruent chords have congruent arcs.
- Set arc RS = arc TV: $5y + 10 = 8y - 14$
- $24 = 3y \rightarrow y = 8$
- Arc RS = 5(8) + 10 = 50°; Arc TV = 8(8) – 14 = 50°. ✓

7. Answer: Each chord = 30 units

- Both chords are equidistant from the center, so they are congruent.
- $\text{half-chord}^2 = r^2 - d^2 = 17^2 - 8^2 = 289 - 64 = 225$
- half-chord = $\sqrt{225} = 15$

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- Full chord = 2 × 15 = 30 units.

8. Answer: $x = 18$, each central angle = 56°

- Theorem: In congruent circles, congruent chords subtend congruent central angles.
- Set the angles equal: $2x + 20 = 4x - 16$
- $36 = 2x \rightarrow x = 18$
- Central angle = $2(18) + 20 = 56°$; check: $4(18) - 16 = 56°$. ✓

9. Answer: Radius = 13 units; RS is also 5 units from the center

- half of PQ = 12; use Pythagorean theorem: $r^2 = 12^2 + 5^2 = 144 + 25 = 169$, so $r = 13$.
- Since arc PQ ≅ arc RS, chord PQ ≅ chord RS (congruent arcs → congruent chords in the same circle).
- Congruent chords are equidistant from the center.
- Therefore RS is also 5 units from the center.

10. Answer: CD = $2\sqrt{(625 - 196)} = 2\sqrt{429} \approx 41.4$ units; angle COD = 112°

- Since arc AB ≅ arc CD in congruent circles, chord AB ≅ chord CD.
- Find AB: half-$AB^2 = 25^2 - 14^2 = 625 - 196 = 429$, so half-AB = $\sqrt{429}$.
- AB = CD = $2\sqrt{429} \approx 41.4$ units.
- Since arc CD ≅ arc AB = 112°, the central angle COD equals the arc measure.
- Therefore angle COD = 112°.
- Also, since CD ≅ AB, CD is the same distance from the center: 14 units (congruent chords in congruent circles are equidistant from their respective centers).

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