

Discrete Random Variables & Probability Distributions

Statistics Worksheet · Grade 10–12

Name: _____

Date: _____

Learning Objectives

- Identify and interpret discrete random variables and their probability distributions
- Calculate probabilities using a probability distribution table, including inequalities and complements
- Construct a probability distribution table from a real-world scenario such as flipping a coin multiple times

Problems

1. The table below shows the probability distribution for the number of pets owned by students in a class. Fill in the missing probability for $X = 2$ so that the distribution is valid.

| X | 0 | 1 | 2 | 3 |
|------|------|------|---|------|
| P(X) | 0.20 | 0.35 | | 0.15 |

2. Using the grade distribution from NCSU Statistics 101 shown below, write in plain words what the probability notation $P(X = 3)$ represents.

| Grade | F | D | C | B | A |
|-------|------|------|------|------|------|
| X | 0 | 1 | 2 | 3 | 4 |
| P(X) | 0.02 | 0.10 | 0.20 | 0.42 | 0.26 |

3. Using the NCSU grade distribution table, find the probability that a randomly selected student earns a grade of B or higher.

| Grade | F | D | C | B | A |
|-------|------|------|------|------|------|
| X | 0 | 1 | 2 | 3 | 4 |
| P(X) | 0.02 | 0.10 | 0.20 | 0.42 | 0.26 |

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4. Using the NCSU grade distribution table, find the probability that a randomly selected student earns a grade worse than a C (i.e., a D or an F).

| Grade | F | D | C | B | A |
|-------|------|------|------|------|------|
| X | 0 | 1 | 2 | 3 | 4 |
| P(X) | 0.02 | 0.10 | 0.20 | 0.42 | 0.26 |

5. Using the NCSU grade distribution table, find the probability that a student earns a grade lower than an A using the complement rule. Show both methods.

| Grade | F | D | C | B | A |
|-------|------|------|------|------|------|
| X | 0 | 1 | 2 | 3 | 4 |
| P(X) | 0.02 | 0.10 | 0.20 | 0.42 | 0.26 |

6. A fair coin is flipped three times and X is the number of heads obtained. List all possible outcomes and complete the probability distribution table below.

| X (# of Heads) | 0 | 1 | 2 | 3 |
|----------------|---|---|---|---|
| P(X) | | | | |

7. Using the coin-flip distribution from Problem 6, find the probability of getting at least 2 heads when flipping a coin three times.

$$P(X \geq 2)$$

8. A survey at a local school asked students how many extracurricular activities they participate in. The results are shown below. Determine whether this is a valid probability distribution and explain why or why not.

| X | 0 | 1 | 2 | 3 | 4 |
|------|------|------|------|------|------|
| P(X) | 0.05 | 0.30 | 0.40 | 0.20 | 0.10 |

9. The number of customers who enter a small coffee shop each hour follows the discrete probability distribution shown. Find the probability that between 2 and 4 customers (inclusive) enter in a given hour.



and also find the probability that fewer than 2 customers enter.

| X | 0 | 1 | 2 | 3 | 4 | 5 |
|----------|----------|----------|----------|----------|----------|----------|
| P(X) | 0.05 | 0.10 | 0.25 | 0.30 | 0.20 | 0.10 |

10. A teacher grades tests on a scale of 0 to 4 (whole numbers only). It is known that: $P(X = 0) = 0.03$, $P(X = 1) = 0.07$, $P(X = 3) = 2$ times $P(X = 2)$, and $P(X = 4) = 0.28$. Find $P(X = 2)$ and $P(X = 3)$, then calculate the probability that a randomly selected student scores at least a 2.

$$P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) = 1$$

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Discrete Random Variables & Probability Distributions — Answer Key

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Answer Key

1. Answer: $P(X = 2) = 0.30$

| X | 0 | 1 | 2 | 3 |
|------|------|------|------|------|
| P(X) | 0.20 | 0.35 | 0.30 | 0.15 |

- All probabilities in a valid distribution must sum to 1.
- $0.20 + 0.35 + P(X=2) + 0.15 = 1$
- $0.70 + P(X=2) = 1$
- $P(X=2) = 1 - 0.70 = 0.30$

2. Answer: The probability that a randomly selected student earns a grade of B.

- X represents the grade point value of the letter grade received.
- X = 3 corresponds to a letter grade of B.
- Therefore $P(X = 3)$ = the probability that a student receives a B.

3. Answer: 0.68

- $P(X \geq 3)$ includes X = 3 and X = 4.
- $P(X = 3) = 0.42$ and $P(X = 4) = 0.26$.
- $P(X \geq 3) = 0.42 + 0.26 = 0.68$.
- About 68% of students earn a B or higher.

4. Answer: 0.12

- Worse than a C means a grade below C, so $X < 2$ (X = 0 or X = 1).
- $P(X = 0) = 0.02$ and $P(X = 1) = 0.10$.
- $P(X < 2) = 0.02 + 0.10 = 0.12$.
- About 12% of students earn a D or F.

5. Answer: 0.74

- Method 1 (direct): $P(X < 4) = P(0) + P(1) + P(2) + P(3) = 0.02 + 0.10 + 0.20 + 0.42 = 0.74$.
- Method 2 (complement): The complement of 'lower than A' is 'getting an A', $P(X=4) = 0.26$.
- $P(X < 4) = 1 - 0.26 = 0.74$.
- Both methods confirm 74% of students earn below an A.

6. Answer: $P(0)=1/8, P(1)=3/8, P(2)=3/8, P(3)=1/8$

| X (# of Heads) | 0 | 1 | 2 | 3 |
|----------------|-----|-----|-----|-----|
| P(X) | 1/8 | 3/8 | 3/8 | 1/8 |

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- All possible outcomes: TTT, TTH, THT, HTT, HHT, HTH, THH, HHH — 8 equally likely outcomes.
- $X=0$: only TTT \rightarrow 1 outcome $\rightarrow P(0)=1/8$.
- $X=1$: TTH, THT, HTT \rightarrow 3 outcomes $\rightarrow P(1)=3/8$.
- $X=2$: HHT, HTH, THH \rightarrow 3 outcomes $\rightarrow P(2)=3/8$.
- $X=3$: only HHH \rightarrow 1 outcome $\rightarrow P(3)=1/8$.
- Check: $1/8 + 3/8 + 3/8 + 1/8 = 8/8 = 1$. ✓

7. Answer: 0.50

- $P(X \geq 2)$ includes $X = 2$ and $X = 3$.
- $P(X = 2) = 3/8$ and $P(X = 3) = 1/8$.
- $P(X \geq 2) = 3/8 + 1/8 = 4/8 = 0.50$.
- There is a 50% chance of getting at least 2 heads.

8. Answer: Valid. All probabilities are between 0 and 1, and they sum to 1.05 — wait, 0.05+0.30+0.40+0.20+0.10 = 1.05. NOT valid.

- Check 1: Each probability must be between 0 and 1. All values satisfy this condition.
- Check 2: All probabilities must sum to exactly 1.
- $0.05 + 0.30 + 0.40 + 0.20 + 0.10 = 1.05 \neq 1$.
- Because the probabilities sum to 1.05, this is NOT a valid probability distribution.

9. Answer: $P(2 \leq X \leq 4) = 0.75$; $P(X < 2) = 0.15$

- Part 1 — $P(2 \leq X \leq 4)$: Add probabilities for $X = 2, 3, 4$.
- $0.25 + 0.30 + 0.20 = 0.75$.
- Part 2 — $P(X < 2)$: Add probabilities for $X = 0$ and $X = 1$.
- $0.05 + 0.10 = 0.15$.
- Verify: 0.75 and 0.15 are both less than 1 and make intuitive sense given the distribution.

10. Answer: $P(X=2) = 0.2067 \approx 0.207$, $P(X=3) = 0.4133 \approx 0.413$; $P(X \geq 2) \approx 0.90$

- Set up the equation: $0.03 + 0.07 + P(2) + P(3) + 0.28 = 1$.
- We know $P(3) = 2 \cdot P(2)$, so substitute: $0.38 + P(2) + 2 \cdot P(2) = 1$.
- $0.38 + 3 \cdot P(2) = 1 \rightarrow 3 \cdot P(2) = 0.62 \rightarrow P(2) = 0.62/3 \approx 0.2067$.
- $P(3) = 2 \times 0.2067 \approx 0.4133$.
- $P(X \geq 2) = P(2) + P(3) + P(4) \approx 0.2067 + 0.4133 + 0.28 = 0.90$.
- About 90% of students score a 2 or higher.

