# Random Variables & Probability Distributions

Statistics Worksheet · Grade 9–12

Name: _____ Date: _____

**Learning Objectives**

- Define random variables and identify their possible values from a given experiment
- Distinguish between discrete and continuous random variables
- Construct a probability distribution table and calculate probabilities for discrete random variables

## Problems

**1.** A fair coin is flipped once. Let X be the number of heads. List all possible values of the random variable X.

$X = ?$

**2.** A single fair die is rolled. Let X be the number showing on top. List all possible values of the random variable X.

$X = ?$

**3.** A coin is flipped three times. Let X be the number of tails obtained. List all possible values of the random variable X and state how many total outcomes are in the sample space.

$X = ?, \quad |S| = ?$

**4.** Two dice are rolled. Let X be the sum of the two numbers facing up. What are the minimum and maximum possible values of X?

$X_{min} = ?, \quad X_{max} = ?$

**5.** A coin is flipped four times. Let X be the number of tails. Complete the probability distribution table below by finding the probability for each value of X. Use the fact that the total sample space has 16 outcomes.

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| X (# of Tails) | Number of Ways | P(X) |
|---|---|---|
| 0 | 1 | |
| 1 | 4 | |
| 2 | 6 | |
| 3 | 4 | |
| 4 | 1 | |

**6.** Using the probability distribution from the four-coin-flip experiment (flipping a coin four times, X = number of tails), find the probability of getting at least 3 tails.

$P(X \geq 3) = ?$

**7.** A bag contains 3 red marbles and 2 blue marbles. Two marbles are drawn one at a time without replacement. Let X be the number of red marbles drawn. List all possible values of X and find P(X = 2).

$P(X = 2) = ?$

**8.** Using the same bag from Problem 7 (3 red, 2 blue marbles, drawing 2 without replacement), complete the full probability distribution table for X = number of red marbles drawn.

| X | P(X) |
|---|---|
| 0 | |
| 1 | |
| 2 | |

**9.** A discrete random variable X has the probability distribution shown below. Find the missing probability P(X = 3) and then calculate the expected value (mean) of X.

| X | P(X) |
|---|---|
| 1 | 0.10 |
| 2 | 0.25 |
| 3 | |

| X | P(X) |
|---|---|
| 4 | 0.20 |
| 5 | 0.15 |

**10.** Two dice are rolled. Let X be the sum of the two numbers facing up. Find P(X = 7) and P(X is even). Express each as a fraction in simplest form. (Hint: There are 36 equally likely outcomes when rolling two dice.)

$$P(X = 7) = ?, \quad P(X \text{ is even}) = ?$$

# Random Variables & Probability Distributions — Answer Key

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## Answer Key

**1. Answer: X = 0, 1**

- When flipping a coin once, the outcomes are Heads (H) or Tails (T).
- If X counts the number of heads: getting T gives X = 0, getting H gives X = 1.
- Therefore, X = {0, 1}.

**2. Answer: X = 1, 2, 3, 4, 5, 6**

- A standard die has faces numbered 1 through 6.
- Each face is a possible outcome, so X can take any value from 1 to 6.
- Therefore, X = {1, 2, 3, 4, 5, 6}.

**3. Answer: X = 0, 1, 2, 3; |S| = 8**

- When flipping a coin 3 times, the total number of outcomes is 2^3 = 8.
- The number of tails can range from 0 (all heads: HHH) to 3 (all tails: TTT).
- Therefore, X = {0, 1, 2, 3} and |S| = 8.

**4. Answer: X_min = 2, X_max = 12**

- The smallest number on each die is 1, so the minimum sum is 1 + 1 = 2.
- The largest number on each die is 6, so the maximum sum is 6 + 6 = 12.
- Therefore, X ranges from 2 to 12.

**5. Answer: See completed table**

| X (# of Tails) | Number of Ways | P(X) |
|---|---|---|
| 0 | 1 | 1/16 |
| 1 | 4 | 4/16 = 1/4 |
| 2 | 6 | 6/16 = 3/8 |
| 3 | 4 | 4/16 = 1/4 |
| 4 | 1 | 1/16 |

- Total sample space = 2^4 = 16 outcomes.
- P(X = 0) = 1/16 (only HHHH has 0 tails).
- P(X = 1) = 4/16 = 1/4 (four ways to get exactly 1 tail).
- P(X = 2) = 6/16 = 3/8 (six ways to get exactly 2 tails).
- P(X = 3) = 4/16 = 1/4 (four ways to get exactly 3 tails).
- P(X = 4) = 1/16 (only TTTT has 4 tails).

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- Check: 1 + 4 + 6 + 4 + 1 = 16, and all probabilities sum to 1.

**6. Answer: $P(X \geq 3) = 5/16$**

- $P(X \geq 3)$ means $P(X = 3) + P(X = 4)$.
- $P(X = 3) = 4/16$ and $P(X = 4) = 1/16$.
- $P(X \geq 3) = 4/16 + 1/16 = 5/16$.

**7. Answer: X = 0, 1, 2; $P(X = 2) = 3/10$**

- X can be 0, 1, or 2 (the number of red marbles drawn out of 2).
- Total ways to choose 2 from 5 marbles: C(5,2) = 10.
- Ways to choose 2 red from 3 red: C(3,2) = 3.
- $P(X = 2) = 3/10$.

**8. Answer: See completed table**

| X | P(X) |
|---|---|
| 0 | 1/10 |
| 1 | 6/10 = 3/5 |
| 2 | 3/10 |

- Total ways to pick 2 marbles from 5: C(5,2) = 10.
- $P(X = 0)$: Choose 0 red and 2 blue: $C(3,0) \times C(2,2) = 1 \times 1 = 1$. $P = 1/10$.
- $P(X = 1)$: Choose 1 red and 1 blue: $C(3,1) \times C(2,1) = 3 \times 2 = 6$. $P = 6/10 = 3/5$.
- $P(X = 2)$: Choose 2 red and 0 blue: $C(3,2) \times C(2,0) = 3 \times 1 = 3$. $P = 3/10$.
- Check: 1/10 + 6/10 + 3/10 = 10/10 = 1. ✓

**9. Answer: $P(X = 3) = 0.30$; $E(X) = 3.05$**

- All probabilities must sum to 1: 0.10 + 0.25 + P(X=3) + 0.20 + 0.15 = 1.
- 0.70 + P(X=3) = 1, so P(X=3) = 0.30.
- $E(X) = 1(0.10) + 2(0.25) + 3(0.30) + 4(0.20) + 5(0.15)$.
- $E(X) = 0.10 + 0.50 + 0.90 + 0.80 + 0.75 = 3.05$.

**10. Answer: $P(X = 7) = 6/36 = 1/6$; $P(X \text{ is even}) = 18/36 = 1/2$**

- Total outcomes when rolling two dice: $6 \times 6 = 36$.
- Ways to get a sum of 7: (1,6),(2,5),(3,4),(4,3),(5,2),(6,1) → 6 ways. $P(X=7) = 6/36 = 1/6$.
- Even sums possible: 2, 4, 6, 8, 10, 12.
- Count outcomes for each: sum=2→1, sum=4→3, sum=6→5, sum=8→5, sum=10→3, sum=12→1. Total = 18.
- $P(X \text{ is even}) = 18/36 = 1/2$.

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