Mean, Variance & Standard Deviation of Random Variables

Statistics Worksheet · Grade 11–12

Name: ________________________________ Date: __________________

Learning Objectives

- Calculate the mean (expected value) of a discrete probability distribution using $E(X) = \Sigma$ x■p■
- Calculate the variance of a discrete probability distribution using $\sigma^2 = \Sigma(x■ - \mu)^2p■$
- Calculate the standard deviation by taking the square root of the variance

Problems

1. A fair die is rolled. The probability distribution is shown below. Find the mean (expected value) of X.

x	P(x)
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

2. A bag contains tickets numbered 0, 1, 2, and 3. The probability distribution below describes drawing one ticket at random. Find the expected value of X.

x	P(x)
0	0.10
1	0.35
2	0.40
3	0.15

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3. A student takes a 3-question multiple-choice quiz. The number of correct answers X has the probability distribution below. Verify that this is a valid probability distribution, then find the mean.

x	P(x)
0	0.20
1	0.35
2	0.30
3	0.15

4. A local bakery records the number of cakes sold per day. The probability distribution is given below. Find the mean number of cakes sold per day and interpret your answer.

x	P(x)	x · P(x)
0	0.05	
1	0.20	
2	0.40	
3	0.25	
4	0.10	

5. Using the probability distribution below, find the variance of X. The mean has already been calculated as 1.5.

x	P(x)
0	0.25
1	0.30
2	0.25
3	0.20

6. A carnival game awards prizes based on the number of rings landed on a post. The probability distribution is given below. Complete the table and find both the mean and the standard deviation of X.

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x	P(x)	x · P(x)	$(x - \mu)^2 \cdot P(x)$
0	0.40		
1	0.30		
2	0.20		
3	0.10		

7. A school raffle sells 500 tickets at $2 each. There is 1 grand prize of $500, 2 second prizes of $100 each, and 5 third prizes of $20 each. Let X be the net gain for a ticket holder. Complete the probability distribution table and find the expected value.

Net Gain (x)	P(x)
$498	1/500
$98	2/500
$18	5/500
−$2	492/500

8. The number of daily customer complaints at a call center is modeled by the probability distribution below. Find the mean, variance, and standard deviation. Round to two decimal places.

x	P(x)
0	0.15
1	0.25
2	0.30
3	0.20
4	0.10

9. Two different investments A and B have the following probability distributions for annual returns (in thousands of dollars). Find the mean and standard deviation for each investment and determine which is the better choice if you prefer lower risk (lower variability).

Return ($000s)	P(Investment A)	P(Investment B)
−2	0.10	0.05
1	0.20	0.15
3	0.40	0.55
5	0.20	0.20
8	0.10	0.05

10. A quality control manager at a factory inspects batches of 5 items. The number of defective items X per batch follows the probability distribution below. Find the mean, variance, and standard deviation. Then determine the probability that a randomly selected batch has more defective items than the mean. Show all work.

x	P(x)	x · P(x)	(x − μ)²	(x − μ)² · P(x)
0	0.33			
1	0.40			
2	0.18			
3	0.06			
4	0.02			
5	0.01			

Mean, Variance & Standard Deviation of Random Variables — Answer Key

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Answer Key

1. Answer: $\mu = 3.5$

- Use the formula μ = Σ x■ · P(x■)
- $\mu = 1(1/6) + 2(1/6) + 3(1/6) + 4(1/6) + 5(1/6) + 6(1/6)$
- $\mu = (1 + 2 + 3 + 4 + 5 + 6) / 6 = 21/6$
- $\mu = 3.5$

2. Answer: $\mu = 1.60$

- Use the formula μ = Σ x■ · P(x■)
- $\mu = 0(0.10) + 1(0.35) + 2(0.40) + 3(0.15)$
- $\mu = 0 + 0.35 + 0.80 + 0.45$
- $\mu = 1.60$

3. Answer: Valid (probabilities sum to 1); $\mu = 1.40$

- Check: 0.20 + 0.35 + 0.30 + 0.15 = 1.00 ✓ Valid distribution
- Use the formula μ = Σ x■ · P(x■)
- $\mu = 0(0.20) + 1(0.35) + 2(0.30) + 3(0.15)$
- $\mu = 0 + 0.35 + 0.60 + 0.45 = 1.40$

4. Answer: $\mu = 2.15$ cakes per day

x	P(x)	x · P(x)
0	0.05	0.00
1	0.20	0.20
2	0.40	0.80
3	0.25	0.75
4	0.10	0.40

- Multiply each x by its probability to fill the third column
- 0(0.05)=0.00, 1(0.20)=0.20, 2(0.40)=0.80, 3(0.25)=0.75, 4(0.10)=0.40
- Sum the third column: $\mu = 0 + 0.20 + 0.80 + 0.75 + 0.40 = 2.15$
- Interpretation: On average, the bakery is expected to sell about 2.15 cakes per day

5. Answer: $\sigma^2 = 1.05$

- Use the formula σ² = Σ (x■ – μ)² · P(x■) where $\mu = 1.5$
- $(0 - 1.5)^2(0.25) = 2.25 \times 0.25 = 0.5625$

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- $(1 - 1.5)^2(0.30) = 0.25 \times 0.30 = 0.075$
- $(2 - 1.5)^2(0.25) = 0.25 \times 0.25 = 0.0625$
- $(3 - 1.5)^2(0.20) = 2.25 \times 0.20 = 0.45$
- $\sigma^2 = 0.5625 + 0.075 + 0.0625 + 0.45 = 1.15$
- Note: Rechecking — $\sigma^2 = 1.15$

6. Answer: $\mu = 1.00$; $\sigma^2 = 0.80$; $\sigma \approx 0.894$

x	P(x)	x · P(x)	(x − μ)² · P(x)
0	0.40	0.00	0.400
1	0.30	0.30	0.000
2	0.20	0.40	0.200
3	0.10	0.30	0.400

- Find μ: $0(0.40)+1(0.30)+2(0.20)+3(0.10) = 0+0.30+0.40+0.30 = 1.00$
- Find σ^2 using $\sigma^2 = \Sigma\ (x■ - \mu)^2 \cdot P(x■)$ with $\mu = 1.00$
- $(0-1)^2(0.40)=0.40$, $(1-1)^2(0.30)=0.00$, $(2-1)^2(0.20)=0.20$, $(3-1)^2(0.10)=0.40$
- $\sigma^2 = 0.40 + 0.00 + 0.20 + 0.40 = 1.00$
- Wait — recomputing: $\sigma^2 = 1.00$; $\sigma = \sqrt{1.00} = 1.00$

7. Answer: E(X) = –\$0.60 (expected loss of \$0.60 per ticket)

- Net gain = Prize – Ticket cost = Prize – \$2
- $E(X) = 498(1/500) + 98(2/500) + 18(5/500) + (-2)(492/500)$
- $E(X) = 0.996 + 0.392 + 0.18 - 1.968$
- $E(X) = 1.568 - 1.968 = -0.40$
- The expected net gain is approximately –\$0.40 per ticket purchased

8. Answer: $\mu = 1.85$; $\sigma^2 = 1.4275$; $\sigma \approx 1.20$

- $\mu = 0(0.15)+1(0.25)+2(0.30)+3(0.20)+4(0.10) = 0+0.25+0.60+0.60+0.40 = 1.85$
- $\sigma^2 = (0-1.85)^2(0.15)+(1-1.85)^2(0.25)+(2-1.85)^2(0.30)+(3-1.85)^2(0.20)+(4-1.85)^2(0.10)$
- $= 3.4225(0.15)+0.7225(0.25)+0.0225(0.30)+1.3225(0.20)+4.6225(0.10)$
- $= 0.5134+0.1806+0.0068+0.2645+0.4623 = 1.4276$
- $\sigma = \sqrt{1.4276} \approx 1.195 \approx 1.20$

9. Answer: Investment A: $\mu = 3.00$, $\sigma \approx 2.28$; Investment B: $\mu = 3.00$, $\sigma \approx 1.73$; Investment B has lower risk

- Investment A: $\mu = -2(0.10)+1(0.20)+3(0.40)+5(0.20)+8(0.10) = -0.2+0.2+1.2+1.0+0.8 = 3.00$
- Investment B: $\mu = -2(0.05)+1(0.15)+3(0.55)+5(0.20)+8(0.05) = -0.1+0.15+1.65+1.0+0.4 = 3.10$
- Investment A $\sigma^2= (-2-3)^2(0.10)+(1-3)^2(0.20)+(3-3)^2(0.40)+(5-3)^2(0.20)+(8-3)^2(0.10) = 2.5+0.8+0+0.8+2.5 = 6.6+0.8 = 5.20$; $\sigma \approx 2.28$
- Investment B $\sigma^2= (-2-3.1)^2(0.05)+(1-3.1)^2(0.15)+(3-3.1)^2(0.55)+(5-3.1)^2(0.20)+(8-3.1)^2(0.05) \approx 1.3005+0.6615+0.0055+0.722+1.2005 = 3.89$; $\sigma \approx 1.97$
- Both investments have similar means. Investment B has the lower standard deviation, indicating lower risk — it is the better choice for a risk-averse investor

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10. Answer: $\mu \approx$ **1.06;** $\sigma^2 \approx$ **0.9364;** $\sigma \approx$ **0.968; P(X > 1.06) = P(X ≥ 2) = 0.27**

x	P(x)	x · P(x)	$(x - \mu)^2$	$(x - \mu)^2 \cdot P(x)$
0	0.33	0.00	1.1236	0.3708
1	0.40	0.40	0.0036	0.0014
2	0.18	0.36	0.8836	0.1590
3	0.06	0.18	3.7636	0.2258
4	0.02	0.08	8.6436	0.1729
5	0.01	0.05	15.5236	0.1552

- Step 1 — Find μ: μ = 0(0.33)+1(0.40)+2(0.18)+3(0.06)+4(0.02)+5(0.01) = 0+0.40+0.36+0.18+0.08+0.05 = 1.07
- Step 2 — Compute $(x■ - \mu)^2$ for each row using $\mu \approx 1.07$
- Step 3 — Compute $(x■ - \mu)^2 \cdot P(x■)$ for each row and sum them
- σ^2 = 0.3708+0.0014+0.1590+0.2258+0.1729+0.1552 ≈ 1.0851; $\sigma = \sqrt{1.0851} \approx 1.042$
- Step 4 — P(X > μ) = P(X > 1.07) = P(X ≥ 2) = 0.18+0.06+0.02+0.01 = 0.27
- Conclusion: There is a 27% chance that a batch has more defective items than the mean

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