Geometric Probability Distribution

Statistics Worksheet · Grade 10–12

Name: ________________________________ Date: ___________________

Learning Objectives

- Identify the conditions of a geometric experiment and distinguish it from a binomial experiment
- Apply the geometric probability formula P(X = x) = p · q^(x–1) to calculate probabilities
- Compute the mean, variance, and standard deviation of a geometric distribution

Problems

1. A fair coin is flipped until a tail appears. Which of the following correctly identifies the conditions that make this a geometric experiment? List all four conditions and explain why 'no fixed number of trials' is the key difference from a binomial experiment.

2. A bag contains 5 red marbles and 15 blue marbles. You draw one marble at a time with replacement until you draw a red marble. Let X be the trial on which the first red marble is drawn. Identify p, q, and write the probability formula for this geometric experiment.

$$P(X = x) = p \cdot q^{x-1}$$

3. A six-sided die is rolled until a 4 appears. What is the probability that the first 4 appears on the third roll?

$$P(X = 3) = p \cdot q^{3-1}$$

4. Suppose 3% of computers in a lab are defective. Spencer checks computers one at a time until he finds a defective one. What is the probability that the first defective computer is the fifth one he checks?

$$P(X = 5) = (0.03)(0.97)^4$$

5. A basketball player makes free throws with a probability of 0.70. What is the probability that her first miss occurs on the second free throw attempt?

$$P(X = 2) = p \cdot q^{2-1}$$

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6. Twenty-five percent of women working at an outlet center have never been married. If women are randomly selected one at a time, what is the probability that the fourth woman selected is the first one who has never been married?

$$P(X=4)=(0.25)(0.75)^3$$

7. Using the same outlet center scenario where 25% of women have never been married, calculate the mean, variance, and standard deviation of the geometric distribution. Round to two decimal places.

$$\mu=\frac{1}{p},\quad \sigma^2=\frac{q}{p^2},\quad \sigma=\sqrt{\frac{q}{p^2}}$$

8. A quality control inspector finds that 8% of items on an assembly line are defective. Items are inspected one by one. Complete the probability distribution table for X = 1, 2, 3, 4, and 5, where X is the trial on which the first defective item is found. Round each probability to four decimal places.

X (trial)	P(X = x)
1	
2	
3	
4	
5	

9. A call center agent successfully resolves a customer complaint with a probability of 0.40 on each call. What is the probability that the agent will need MORE than 3 calls to resolve the first complaint? Hint: Use $P(X > 3) = 1 – P(X \leq 3)$.

$$P(X>3)=1-[P(X=1)+P(X=2)+P(X=3)]$$

10. A medical test correctly identifies a rare disease with a probability of 0.05. Patients are tested one at a time until the first positive result. (a) Find the probability that exactly the 10th patient tests positive. (b) Find the mean and standard deviation of this distribution. (c) If a clinic tests 30 patients and gets no positive result, is this surprising based on the mean? Explain using your computed values.

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$$P(X = 10) = (0.05)(0.95)^9$$

Geometric Probability Distribution — Answer Key

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Answer Key

1. Answer: No fixed number of trials; two outcomes (H or T); independent trials; constant $p = 0.5$ each flip.

- Condition 1: No fixed number of trials — you flip until you get a tail, so trials are not predetermined.
- Condition 2: Two outcomes — Head (failure) or Tail (success).
- Condition 3: Trials are independent — each flip does not affect the next.
- Condition 4: Probability of success is constant — $p = 0.5$ every flip.
- Key difference from binomial: binomial has a fixed n; geometric does not.

2. Answer: $p = 0.25$, $q = 0.75$, $P(X = x) = 0.25 \cdot (0.75)^{(x-1)}$

- Total marbles = 5 + 15 = 20.
- $p = 5/20 = 0.25$ (probability of drawing red).
- $q = 1 - 0.25 = 0.75$ (probability of not drawing red).
- Formula: $P(X = x) = 0.25 \cdot (0.75)^{(x-1)}$.

3. Answer: $P(X = 3) \approx 0.1157$ or about 11.57%

- $p = 1/6 \approx 0.1667$ (probability of rolling a 4).
- $q = 5/6 \approx 0.8333$ (probability of not rolling a 4).
- Apply formula: $P(X = 3) = (1/6) \cdot (5/6)^2$.
- $P(X = 3) = (1/6) \cdot (25/36) = 25/216 \approx 0.1157$.

4. Answer: $P(X = 5) \approx 0.0266$ or about 2.66%

- $p = 0.03$, $q = 0.97$, $X = 5$.
- Apply formula: $P(X = 5) = (0.03)(0.97)^{(5-1)} = (0.03)(0.97)^4$.
- $(0.97)^4 \approx 0.8853$.
- $P(X = 5) \approx 0.03 \times 0.8853 \approx 0.0266$.

5. Answer: $P(X = 2) = 0.21$ or 21%

- Let success = missing a free throw, so $p = 1 - 0.70 = 0.30$.
- $q = 1 - 0.30 = 0.70$.
- $P(X = 2) = (0.30)(0.70)^{(2-1)} = (0.30)(0.70)^1$.
- $P(X = 2) = 0.30 \times 0.70 = 0.21$.

6. Answer: $P(X = 4) \approx 0.1055$ or about 10.55%

- $p = 0.25$ (never married), $q = 0.75$, $X = 4$.
- $P(X = 4) = (0.25)(0.75)^{(4-1)} = (0.25)(0.75)^3$.
- $(0.75)^3 = 0.421875$.
- $P(X = 4) = 0.25 \times 0.421875 \approx 0.1055$.

7. Answer: $\mu = 4$, $\sigma^2 = 12$, $\sigma \approx 3.46$

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- $p = 0.25$, $q = 0.75$.
- Mean: $\mu = 1/p = 1/0.25 = 4$.
- Variance: $\sigma^2 = q/p^2 = 0.75/(0.25)^2 = 0.75/0.0625 = 12$.
- Standard Deviation: $\sigma = \sqrt{12} \approx 3.46$.

8. Answer: See completed table

X (trial)	P(X = x)
1	0.0800
2	0.0736
3	0.0677
4	0.0623
5	0.0573

- $p = 0.08$, $q = 0.92$. Use $P(X = x) = (0.08)(0.92)^{(x-1)}$.
- $P(X = 1) = 0.08 \times (0.92)^0 = 0.0800$.
- $P(X = 2) = 0.08 \times (0.92)^1 = 0.08 \times 0.92 = 0.0736$.
- $P(X = 3) = 0.08 \times (0.92)^2 = 0.08 \times 0.8464 = 0.0677$.
- $P(X = 4) = 0.08 \times (0.92)^3 = 0.08 \times 0.7787 = 0.0623$.
- $P(X = 5) = 0.08 \times (0.92)^4 = 0.08 \times 0.7164 = 0.0573$.

9. Answer: $P(X > 3) = 0.216$ or 21.6%

- $p = 0.40$, $q = 0.60$.
- $P(X = 1) = 0.40 \times (0.60)^0 = 0.40$.
- $P(X = 2) = 0.40 \times (0.60)^1 = 0.24$.
- $P(X = 3) = 0.40 \times (0.60)^2 = 0.40 \times 0.36 = 0.144$.
- $P(X \leq 3) = 0.40 + 0.24 + 0.144 = 0.784$.
- $P(X > 3) = 1 - 0.784 = 0.216$.

10. Answer: $P(X=10) \approx 0.0315$; $\mu = 20$, $\sigma \approx 19.49$; testing 30 with no positive is within ~0.5 standard deviations above the mean, so not extremely surprising.

- $p = 0.05$, $q = 0.95$.
- (a) $P(X = 10) = (0.05)(0.95)^9 = 0.05 \times 0.6302 \approx 0.0315$.
- (b) Mean: $\mu = 1/p = 1/0.05 = 20$ patients.
- Variance: $\sigma^2 = q/p^2 = 0.95/(0.05)^2 = 0.95/0.0025 = 380$.
- Standard Deviation: $\sigma = \sqrt{380} \approx 19.49$.
- (c) The mean is 20, meaning on average the first positive occurs on the 20th patient. Testing 30 with no positive is only about 0.5 standard deviations above the mean ($z \approx (30-20)/19.49 \approx 0.51$), so this outcome is not particularly surprising.

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