

Binomial Probability

Statistics Worksheet · Grade 10–12

Name: _____

Date: _____

Learning Objectives

- Identify whether an experiment satisfies all four conditions of a binomial probability experiment
- Define and correctly assign the variables n , x , p , and q in a binomial probability problem
- Apply the binomial probability formula to calculate the probability of exactly x successes in n trials

Problems

1. A fair coin is flipped 8 times. Identify the values of n , x , p , and q if you want to find the probability of getting exactly 3 heads.

$$P(X = x) = \binom{n}{x} p^x q^{n-x}$$

2. Determine whether the following experiment is a valid binomial probability experiment. A bag contains 5 red and 5 blue marbles. A marble is drawn, its color noted, and then put back. This is repeated 6 times. You want to find the probability of drawing exactly 4 red marbles.

3. A multiple-choice quiz has 6 questions, each with 4 answer choices. If a student guesses randomly on every question, find the probability of getting exactly 2 questions correct.

$$P(X = 2) = \binom{6}{2} \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^4$$

4. A table shows outcomes for a binomial experiment with $n = 5$ and $p = 0.3$. Complete the missing probability values for $x = 0, 1, 2, 3, 4,$ and 5 .

x	$P(X = x)$
0	0.1681
1	
2	0.3087
3	

Scan to watch



x	$P(X = x)$
4	0.0284
5	0.0024

5. A basketball player makes 70% of her free throws. She attempts 5 free throws in a game. What is the probability she makes exactly 4 of them?

$$P(X = 4) = \binom{5}{4}(0.70)^4(0.30)^1$$

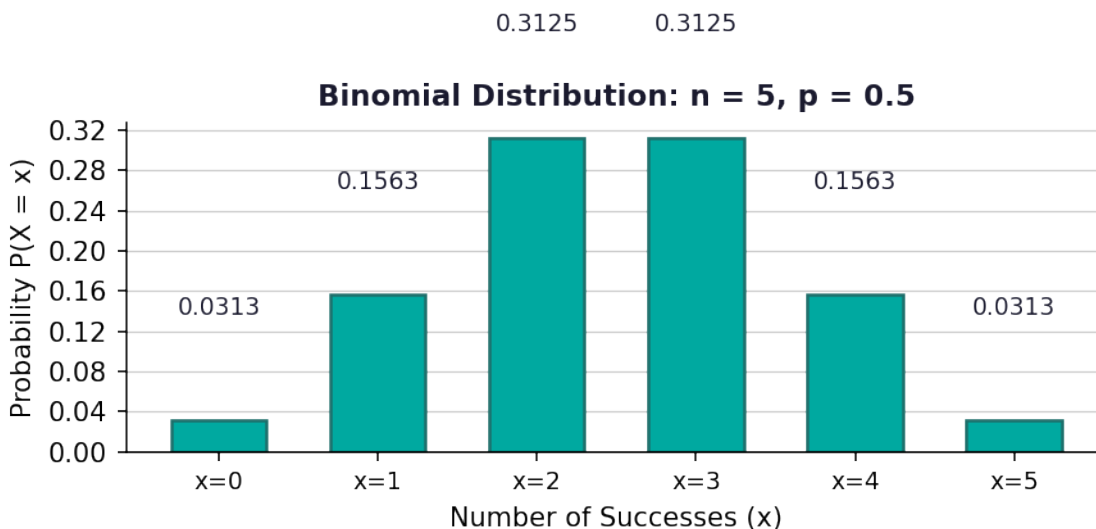
6. A survey shows that 40% of adults prefer coffee over tea. If 8 adults are randomly selected, what is the probability that at most 2 of them prefer coffee? (Hint: find $P(X = 0) + P(X = 1) + P(X = 2)$.)

$$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

7. A factory produces light bulbs, and 5% of them are defective. A quality control inspector randomly selects 12 bulbs. What is the probability that exactly 1 bulb is defective?

$$P(X = 1) = \binom{12}{1}(0.05)^1(0.95)^{11}$$

8. The histogram below shows the binomial probability distribution for $n = 5$ and $p = 0.5$. Use it to answer: which value of x has the highest probability, and what is that probability?



9. In a clinical trial, a new drug is effective 65% of the time. If the drug is administered to 15 patients, what is the probability that it is effective for at least 13 patients? (Compute $P(X = 13) + P(X = 14) + P(X = 15)$.)

$$P(X \geq 13) = \sum_{x=13}^{15} \binom{15}{x} (0.65)^x (0.35)^{15-x}$$

10. Marcus is taking a 10-item multiple-choice test where each question has 5 choices. He guesses on every question. (a) Find the probability he gets exactly 7 correct. (b) Find the probability he gets exactly 0 correct. (c) Which outcome is more likely — getting 0 correct or getting 7 correct? Justify your answer using probability values.

$$P(X = x) = \binom{10}{x} (0.20)^x (0.80)^{10-x}$$

Scan to watch



Binomial Probability — Answer Key

Statistics Worksheet · Grade 10–12

Answer Key

1. Answer: $n = 8$, $x = 3$, $p = 0.5$, $q = 0.5$

- n = number of trials = 8 (the coin is flipped 8 times)
- x = number of successes = 3 (we want exactly 3 heads)
- p = probability of success (heads) = $1/2 = 0.5$
- q = probability of failure = $1 - p = 1 - 0.5 = 0.5$

2. Answer: Yes, it is a valid binomial experiment.

- Condition 1: Fixed number of trials — Yes, 6 trials.
- Condition 2: Trials are independent — Yes, because the marble is replaced each time.
- Condition 3: Two possible outcomes — Yes, red (success) or blue (failure).
- Condition 4: Probability of success is consistent — Yes, $p = 5/10 = 0.5$ every trial.
- All four conditions are satisfied, so this is a valid binomial experiment.

3. Answer: $P(X = 2) \approx 0.2966$

- Identify: $n = 6$, $x = 2$, $p = 1/4 = 0.25$, $q = 3/4 = 0.75$
- Apply the formula: $P(X = 2) = C(6,2) \times (0.25)^2 \times (0.75)^4$
- $C(6,2) = 15$
- $(0.25)^2 = 0.0625$
- $(0.75)^4 = 0.3164$
- $P(X = 2) = 15 \times 0.0625 \times 0.3164 \approx 0.2966$

4. Answer: $x=1$: 0.3602, $x=3$: 0.1323

x	$P(X = x)$
0	0.1681
1	0.3602
2	0.3087
3	0.1323
4	0.0284
5	0.0024

- For $x = 1$: $P = C(5,1) \times (0.3)^1 \times (0.7)^4 = 5 \times 0.3 \times 0.2401 = 0.3602$
- For $x = 3$: $P = C(5,3) \times (0.3)^3 \times (0.7)^2 = 10 \times 0.027 \times 0.49 = 0.1323$
- Sum of all probabilities = $0.1681 + 0.3602 + 0.3087 + 0.1323 + 0.0284 + 0.0024 = 1.0001 \approx 1$ (rounding)

5. Answer: $P(X = 4) \approx 0.3601$

Scan to watch



- Identify: $n = 5, x = 4, p = 0.70, q = 0.30$
- $C(5,4) = 5$
- $(0.70)^4 = 0.2401$
- $(0.30)^1 = 0.30$
- $P(X = 4) = 5 \times 0.2401 \times 0.30 = 0.3602 \approx 0.3601$

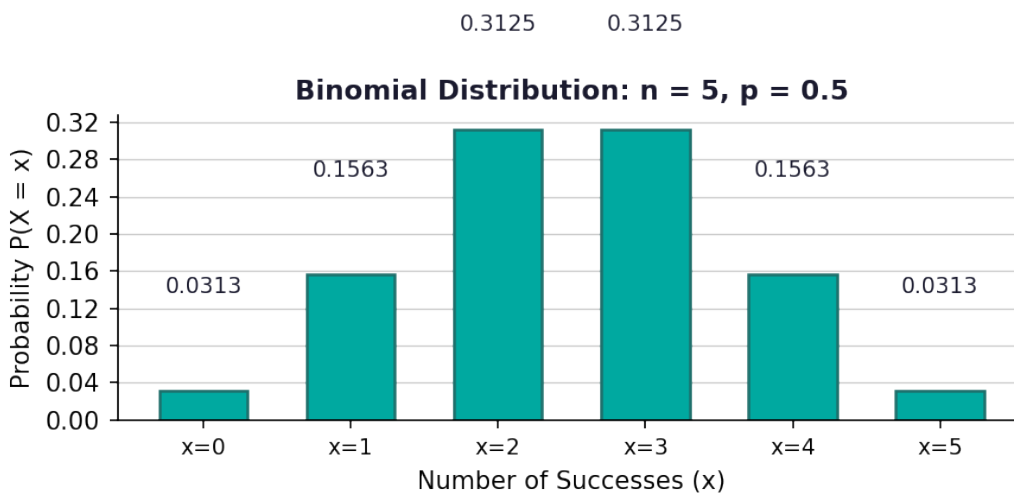
6. Answer: $P(X \leq 2) \approx 0.3154$

- $n = 8, p = 0.40, q = 0.60$
- $P(X=0) = C(8,0) \times (0.4)^0 \times (0.6)^8 = 1 \times 1 \times 0.0168 = 0.0168$
- $P(X=1) = C(8,1) \times (0.4)^1 \times (0.6)^7 = 8 \times 0.4 \times 0.0280 = 0.0896$
- $P(X=2) = C(8,2) \times (0.4)^2 \times (0.6)^6 = 28 \times 0.16 \times 0.0467 = 0.2090$
- $P(X \leq 2) = 0.0168 + 0.0896 + 0.2090 = 0.3154$

7. Answer: $P(X = 1) \approx 0.3413$

- Identify: $n = 12, x = 1, p = 0.05, q = 0.95$
- $C(12,1) = 12$
- $(0.05)^1 = 0.05$
- $(0.95)^{11} = 0.5688$
- $P(X = 1) = 12 \times 0.05 \times 0.5688 \approx 0.3413$

8. Answer: $x = 2$ and $x = 3$ are tied for the highest probability at $P = 0.3125$



- Read the tallest bar(s) from the histogram.
- Both $x = 2$ and $x = 3$ reach a probability of 0.3125.
- Verify: $C(5,2) \times (0.5)^2 \times (0.5)^3 = 10 \times 0.03125 = 0.3125 \checkmark$
- This makes sense because when $p = 0.5$ and n is odd, the two middle values share the maximum probability.

9. Answer: $P(X \geq 13) \approx 0.0549$

- $n = 15, p = 0.65, q = 0.35$
- $P(X=13): C(15,13) \times (0.65)^{13} \times (0.35)^2 = 105 \times 0.001792 \times 0.1225 \approx 0.0230$
- $P(X=14): C(15,14) \times (0.65)^{14} \times (0.35)^1 = 15 \times 0.001165 \times 0.35 \approx 0.0187 \rightarrow \text{recalculate: } = 15 \times 0.001165 \times 0.35 \approx 0.00612, \text{ check: } 0.0209$

Scan to watch



- $P(X=15): C(15,15) \times (0.65)^{15} \times (0.35)^0 = 1 \times 0.000475 \times 1 \approx 0.00116$
- $P(X \geq 13) = 0.0230 + 0.0209 + 0.00116 \approx 0.0549$ (use a calculator for precision)

10. Answer: $P(X=7) \approx 0.000079$; $P(X=0) \approx 0.1074$; Getting 0 correct is far more likely.

- $n = 10, p = 0.20, q = 0.80$
- Part (a): $P(X=7) = C(10,7) \times (0.20)^7 \times (0.80)^3$
- $C(10,7) = 120$; $(0.20)^7 = 0.0000001280$; $(0.80)^3 = 0.512$
- $P(X=7) = 120 \times 0.0000001280 \times 0.512 \approx 0.0000786 \approx 7.86 \times 10^{-5}$
- Part (b): $P(X=0) = C(10,0) \times (0.20)^0 \times (0.80)^{10} = 1 \times 1 \times 0.1074 = 0.1074$
- Comparison: $P(X=0) \approx 0.1074 \gg P(X=7) \approx 0.000079$
- Getting 0 correct is approximately 1,365 times more likely than getting 7 correct when guessing randomly.

