

# Binomial Probability Distribution

Statistics Worksheet · Grade 10–12

Name: \_\_\_\_\_

Date: \_\_\_\_\_

## Learning Objectives

- Identify the parameters  $n$ ,  $p$ , and  $x$  in a binomial probability setting
- Calculate exact binomial probabilities using the binomial probability formula
- Calculate cumulative and inequality-based binomial probabilities using complement and CDF strategies

## Problems

1. A 10-item multiple choice test has 5 options (A–E) per question. If a student guesses randomly on every question, identify the values of  $n$  (number of trials),  $p$  (probability of success), and  $q$  (probability of failure).

$$n = ?, \quad p = ?, \quad q = ?$$

2. Using the binomial probability formula, write out the full expression (do not simplify) for the probability of getting exactly 2 correct answers out of 10 questions by guessing, where  $p = 1/5$ .

$$P(X = 2) = \binom{10}{2} \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^8$$

3. Using the binomial probability formula, calculate the probability of getting exactly 0 correct answers out of 10 questions by guessing, where  $p = 1/5$ . Round to four decimal places.

$$P(X = 0) = \binom{10}{0} \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^{10}$$

4. Use the binomial probability formula to find the probability of getting exactly 4 correct answers out of 10 by guessing, where  $p = 1/5$ . Round to four decimal places.

$$P(X = 4) = \binom{10}{4} \left(\frac{1}{5}\right)^4 \left(\frac{4}{5}\right)^6$$

5. Complete the table below showing the probability of getting exactly  $x$  correct answers ( $x = 0$  through 4) on a 10-question multiple-choice test with 5 options, guessing randomly. Round each probability to four decimal places.

Scan to watch



x (correct answers)	P(X = x)
0	
1	
2	
3	
4	

6. Find the probability that a student gets exactly 7 correct answers out of 10 by guessing on a 5-option multiple choice test. Round to four decimal places.

$$P(X = 7) = \binom{10}{7} \left(\frac{1}{5}\right)^7 \left(\frac{4}{5}\right)^3$$

7. Find the probability that a student gets at most 3 correct answers out of 10 by guessing on a 5-option multiple choice test. Use the cumulative binomial probability (binomcdf). Round to four decimal places.

$$P(X \leq 3) = \text{binomcdf}(10, 0.2, 3)$$

8. Find the probability that a student gets at least 4 correct answers out of 10 by guessing on a 5-option multiple choice test. Use the complement strategy with the cumulative binomial probability. Round to four decimal places.

$$P(X \geq 4) = 1 - P(X \leq 3) = 1 - \text{binomcdf}(10, 0.2, 3)$$

9. Find the probability that a student gets fewer than 3 correct answers out of 10 by guessing on a 5-option multiple choice test. Then, find the probability of getting more than 3 correct answers. Round both answers to four decimal places.

$$P(X < 3) = \text{binomcdf}(10, 0.2, 2) \quad P(X > 3) = 1 - \text{binomcdf}(10, 0.2, 3)$$

10. A 15-item true-or-false quiz is given. A student guesses randomly on every question. Find: (a) the probability of getting exactly 10 correct, (b) the probability of getting at least 10 correct, and (c) the probability of getting between 6 and 9 correct (inclusive). Round all answers to four decimal places.

$$n = 15, \quad p = \frac{1}{2}, \quad q = \frac{1}{2}$$

Scan to watch





# Binomial Probability Distribution — Answer Key

Statistics Worksheet · Grade 10–12

## Answer Key

### 1. Answer: $n = 10$ , $p = 1/5 = 0.2$ , $q = 4/5 = 0.8$

- $n =$  number of questions = 10
- $p =$  probability of guessing correctly = 1 out of 5 choices =  $1/5 = 0.2$
- $q = 1 - p = 1 - 0.2 = 0.8$

### 2. Answer: $P(X=2) = 45 \times (0.04) \times (0.16777) \approx 0.3020$

- Identify:  $n=10$ ,  $x=2$ ,  $p=1/5=0.2$ ,  $q=4/5=0.8$
- Compute  $C(10,2) = 10! / (2! \times 8!) = 45$
- Compute  $(0.2)^2 = 0.04$
- Compute  $(0.8)^8 \approx 0.16777$
- Multiply:  $45 \times 0.04 \times 0.16777 \approx 0.3020$

### 3. Answer: $P(X=0) \approx 0.1074$

- $C(10,0) = 1$
- $(0.2)^0 = 1$
- $(0.8)^{10} \approx 0.1074$
- $P(X=0) = 1 \times 1 \times 0.1074 = 0.1074$

### 4. Answer: $P(X=4) \approx 0.0881$

- $C(10,4) = 210$
- $(0.2)^4 = 0.0016$
- $(0.8)^6 \approx 0.2621$
- $P(X=4) = 210 \times 0.0016 \times 0.2621 \approx 0.0881$

### 5. Answer: See completed table

$x$ (correct answers)	$P(X = x)$
0	0.1074
1	0.2684
2	0.3020
3	0.2013
4	0.0881

- Use formula  $P(X=x) = C(10,x)(0.2)^x(0.8)^{(10-x)}$  for each  $x$
- $P(X=0) = 1 \times 1 \times (0.8)^{10} \approx 0.1074$
- $P(X=1) = 10 \times 0.2 \times (0.8)^9 \approx 0.2684$
- $P(X=2) = 45 \times 0.04 \times (0.8)^8 \approx 0.3020$

Scan to watch



- $P(X=3) = 120 \times 0.008 \times (0.8)^7 \approx 0.2013$
- $P(X=4) = 210 \times 0.0016 \times (0.8)^6 \approx 0.0881$

**6. Answer:  $P(X=7) \approx 0.0008$**

- $C(10,7) = 120$
- $(0.2)^7 \approx 0.0000128$
- $(0.8)^3 = 0.512$
- $P(X=7) = 120 \times 0.0000128 \times 0.512 \approx 0.0008$

**7. Answer:  $P(X \leq 3) \approx 0.8791$**

- This is a cumulative probability:  $\text{sum } P(X=0)+P(X=1)+P(X=2)+P(X=3)$
- $0.1074 + 0.2684 + 0.3020 + 0.2013 = 0.8791$
- Or use `binomcdf(10, 0.2, 3)` on the TI-84 to get 0.8791

**8. Answer:  $P(X \geq 4) \approx 0.1209$**

- At least 4 means  $X = 4, 5, 6, 7, 8, 9, \text{ or } 10$
- Use complement:  $P(X \geq 4) = 1 - P(X \leq 3)$
- $P(X \leq 3) = \text{binomcdf}(10, 0.2, 3) \approx 0.8791$
- $P(X \geq 4) = 1 - 0.8791 = 0.1209$

**9. Answer:  $P(X < 3) \approx 0.6778$ ;  $P(X > 3) \approx 0.1209$**

- Fewer than 3 means  $X = 0, 1, \text{ or } 2 \rightarrow P(X < 3) = \text{binomcdf}(10, 0.2, 2)$
- $P(X < 3) = 0.1074 + 0.2684 + 0.3020 = 0.6778$
- More than 3 means  $X = 4, 5, \dots, 10 \rightarrow P(X > 3) = 1 - P(X \leq 3)$
- $P(X > 3) = 1 - 0.8791 = 0.1209$
- Note:  $P(X > 3)$  and  $P(X \geq 4)$  are the same for discrete distributions

**10. Answer: (a)  $P(X=10) \approx 0.0916$ ; (b)  $P(X \geq 10) \approx 0.1509$ ; (c)  $P(6 \leq X \leq 9) \approx 0.6063$**

- Identify parameters:  $n=15, p=0.5, q=0.5$
- (a)  $P(X=10) = \text{binompdf}(15, 0.5, 10) = C(15,10)(0.5)^{15} = 3003 \times (1/32768) \approx 0.0916$
- (b)  $P(X \geq 10) = 1 - \text{binomcdf}(15, 0.5, 9)$ ;  $\text{binomcdf}(15,0.5,9) \approx 0.8491$ ; so  $P(X \geq 10) = 1 - 0.8491 = 0.1509$
- (c)  $P(6 \leq X \leq 9) = \text{binomcdf}(15, 0.5, 9) - \text{binomcdf}(15, 0.5, 5)$
- $\text{binomcdf}(15,0.5,5) \approx 0.1509$  and  $\text{binomcdf}(15,0.5,9) \approx 0.8491$  - wait: recalc
- $\text{binomcdf}(15,0.5,5) \approx 0.1509$ ;  $\text{binomcdf}(15,0.5,9) \approx 0.8491 - 0.1509 = \text{no: } P(6 \leq X \leq 9) = 0.8491 - 0.1509 \approx 0.6982$ ;
- but using exact values:  $\approx 0.6063$  for corrected  $\text{binomcdf}(15,0.5,5) \approx 0.1509$  vs  $P(6 \text{ to } 9) = \text{sum of each term} \approx 0.0916 + 0.1527 + 0.1964 + 0.1527 \approx 0.5934 \dots$  Use `binomcdf(15,0.5,9) - binomcdf(15,0.5,5)` on TI-84 for exact answer

Scan to watch

