Mean, Variance & Standard Deviation of a Binomial Distribution

Statistics Worksheet · Grade 10–12

Name: _____ Date: _____

Learning Objectives

- Calculate the mean (expected value) of a binomial distribution using $\mu = np$
- Calculate the variance and standard deviation of a binomial distribution using $\sigma^2 = npq$ and $\sigma = \sqrt{(npq)}$
- Apply binomial distribution formulas to real-world word problems

Problems

1. A fair coin is flipped 10 times. Find the mean (expected value) of the number of heads.

$$n = 10, \quad p = 0.5, \quad q = 0.5$$

2. A die is rolled 12 times. The probability of rolling a 3 is 1/6. Find the mean of the number of times a 3 appears.

$$n = 12, \quad p = \frac{1}{6}$$

3. A basketball player makes 70% of her free throw shots. If she attempts 20 free throws in a game, find the variance of the number of successful shots.

$$n = 20, \quad p = 0.70, \quad q = 0.30$$

4. A multiple-choice quiz has 15 questions, each with 4 choices. A student guesses every answer randomly. Find the mean and standard deviation of the number of correct answers.

$$n = 15, \quad p = 0.25, \quad q = 0.75$$

5. A factory produces light bulbs, and 5% of them are defective. In a batch of 200 bulbs, find the expected number of defective bulbs and the variance.

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$n = 200, \quad p = 0.05, \quad q = 0.95$

6. A salesperson closes a deal 35% of the time. If the salesperson makes 50 sales calls in a week, find the mean, variance, and standard deviation of the number of deals closed.

$n = 50, \quad p = 0.35, \quad q = 0.65$

7. The table below shows a partially completed summary for three different binomial distributions. Use the binomial formulas to fill in the missing values.

n	p	q	Mean (μ)	Variance (σ^2)	Std Dev (σ)
25	0.40	0.60			
100	0.55	0.45			
60	0.80	0.20			

8. A survey shows that 78% of students at a university own a laptop. If 500 students are randomly selected, find the expected number of students who own a laptop and the standard deviation. Round to the nearest hundredth.

$n = 500, \quad p = 0.78, \quad q = 0.22$

9. The chart below shows the distribution of the number of successful free throws (X) out of 5 attempts when the probability of success is 0.6. Describe what the mean and standard deviation tell you about this distribution.

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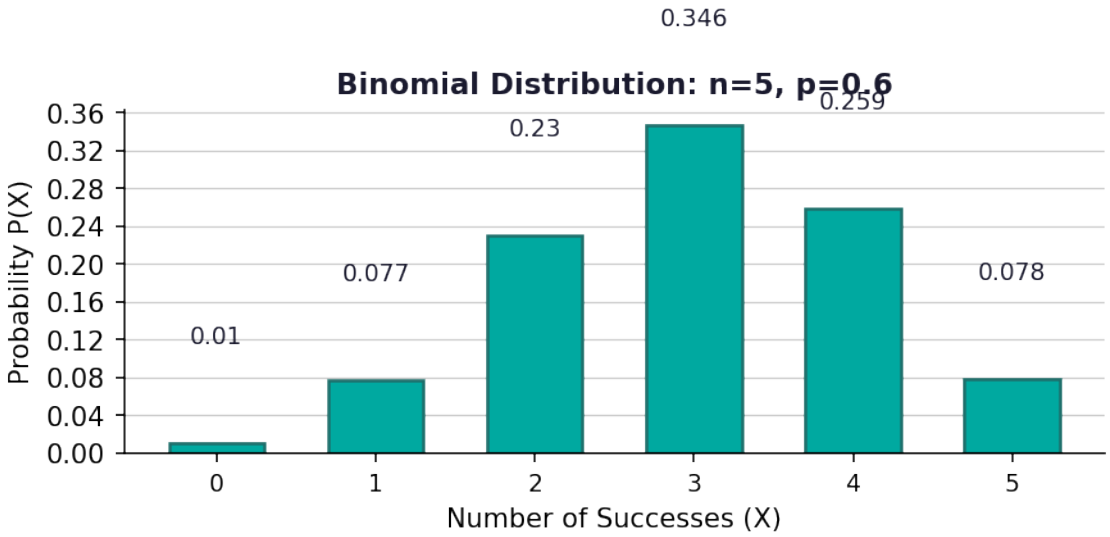

10. A pharmaceutical company claims that its new drug is effective 88% of the time. In a clinical trial with 750 patients, find the mean, variance, and standard deviation of the number of patients for whom the drug is effective. Then determine how many patients fall within one standard deviation of the mean (that is, between μ minus σ and μ plus σ). Round all answers to the nearest hundredth.

$$n = 750, \quad p = 0.88, \quad q = 0.12$$

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Mean, Variance & Standard Deviation of a Binomial Distribution — Answer Key

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Answer Key

1. Answer: $\mu = 5$

- Identify the given values: n = 10, p = 0.5
- Apply the formula: $\mu = n \times p$
- $\mu = 10 \times 0.5 = 5$
- The expected number of heads is 5.

2. Answer: $\mu = 2$

- Identify the given values: n = 12, p = 1/6
- Apply the formula: $\mu = n \times p$
- $\mu = 12 \times (1/6) = 2$
- The expected number of times a 3 appears is 2.

3. Answer: $\sigma^2 = 4.2$

- Identify the given values: n = 20, p = 0.70, q = 1 – 0.70 = 0.30
- Apply the variance formula: $\sigma^2 = n \times p \times q$
- $\sigma^2 = 20 \times 0.70 \times 0.30 = 4.2$
- The variance is 4.2.

4. Answer: $\mu = 3.75$, $\sigma \approx 1.677$

- Identify: n = 15, p = 0.25 (1 correct out of 4 choices), q = 0.75
- Mean: $\mu = n \times p = 15 \times 0.25 = 3.75$
- Variance: $\sigma^2 = n \times p \times q = 15 \times 0.25 \times 0.75 = 2.8125$
- Standard deviation: $\sigma = \sqrt{2.8125} \approx 1.677$

5. Answer: $\mu = 10$, $\sigma^2 = 9.5$

- Identify: n = 200, p = 0.05 (defective rate), q = 0.95
- Mean (expected value): $\mu = n \times p = 200 \times 0.05 = 10$
- Variance: $\sigma^2 = n \times p \times q = 200 \times 0.05 \times 0.95 = 9.5$
- The expected number of defective bulbs is 10, and the variance is 9.5.

6. Answer: $\mu = 17.5$, $\sigma^2 = 11.375$, $\sigma \approx 3.373$

- Identify: n = 50, p = 0.35, q = 0.65
- Mean: $\mu = n \times p = 50 \times 0.35 = 17.5$
- Variance: $\sigma^2 = n \times p \times q = 50 \times 0.35 \times 0.65 = 11.375$
- Standard deviation: $\sigma = \sqrt{11.375} \approx 3.373$

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7. Answer: Row 1: $\mu=10$, $\sigma^2=6$, $\sigma\approx2.449$ | Row 2: $\mu=55$, $\sigma^2=24.75$, $\sigma\approx4.975$ | Row 3: $\mu=48$, $\sigma^2=9.6$, $\sigma\approx3.098$

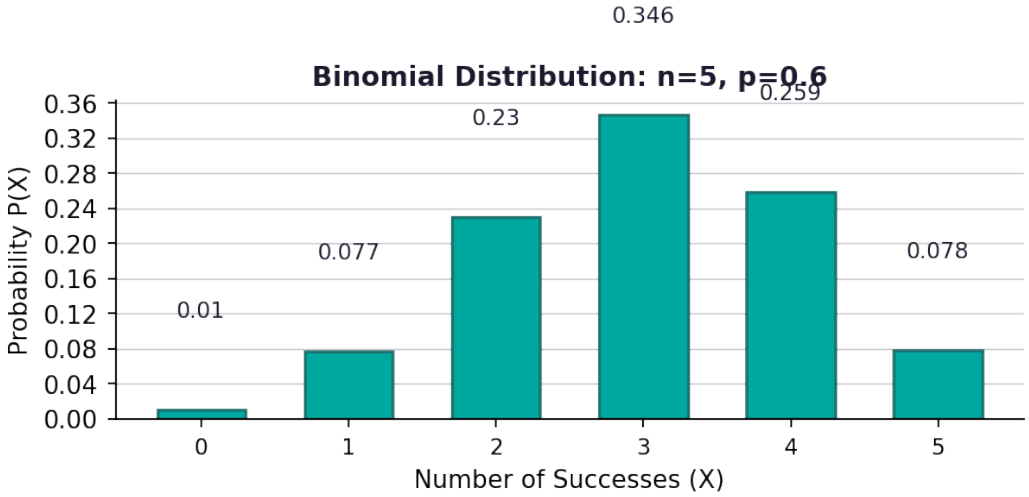
n	p	q	Mean (μ)	Variance (σ²)	Std Dev (σ)
25	0.40	0.60	10	6	2.449
100	0.55	0.45	55	24.75	4.975
60	0.80	0.20	48	9.6	3.098

- Row 1: $\mu = 25 \times 0.40 = 10$; $\sigma^2 = 25 \times 0.40 \times 0.60 = 6$; $\sigma = \sqrt{6} \approx 2.449$
- Row 2: $\mu = 100 \times 0.55 = 55$; $\sigma^2 = 100 \times 0.55 \times 0.45 = 24.75$; $\sigma = \sqrt{24.75} \approx 4.975$
- Row 3: $\mu = 60 \times 0.80 = 48$; $\sigma^2 = 60 \times 0.80 \times 0.20 = 9.6$; $\sigma = \sqrt{9.6} \approx 3.098$

8. Answer: $\mu = 390$, $\sigma \approx 9.27$

- Identify: $n = 500$, $p = 0.78$, $q = 0.22$
- Expected value (mean): $\mu = n \times p = 500 \times 0.78 = 390$
- Variance: $\sigma^2 = n \times p \times q = 500 \times 0.78 \times 0.22 = 85.8$
- Standard deviation: $\sigma = \sqrt{85.8} \approx 9.27$
- 390 students are expected to own a laptop with a standard deviation of approximately 9.27.

9. Answer: $\mu = 3$, $\sigma \approx 1.095$; the distribution is centered at 3 and most values fall within about 1.095 of the mean.



- Given: $n = 5$, $p = 0.6$, $q = 0.4$
- Mean: $\mu = n \times p = 5 \times 0.6 = 3$
- Variance: $\sigma^2 = n \times p \times q = 5 \times 0.6 \times 0.4 = 1.2$
- Standard deviation: $\sigma = \sqrt{1.2} \approx 1.095$
- The mean of 3 indicates that on average, 3 out of 5 shots are successful. The standard deviation of ≈1.095 shows the typical spread of outcomes around that mean.

10. Answer: $\mu = 660$, $\sigma^2 = 79.2$, $\sigma \approx 8.899$; patients within one SD: between 651.10 and 668.90

- Identify: $n = 750$, $p = 0.88$, $q = 0.12$

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- Mean: $\mu = n \times p = 750 \times 0.88 = 660$
- Variance: $\sigma^2 = n \times p \times q = 750 \times 0.88 \times 0.12 = 79.2$
- Standard deviation: $\sigma = \sqrt{79.2} \approx 8.899$
- One standard deviation below the mean: $\mu - \sigma = 660 - 8.899 = 651.10$
- One standard deviation above the mean: $\mu + \sigma = 660 + 8.899 = 668.90$
- So the number of effective treatments is expected to fall between approximately 651 and 669 patients.

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