Sampling Distribution of Sample Proportions

Statistics Worksheet · Grade 11–12 / Introductory College Statistics

Name: ________________________________ Date: __________________

Learning Objectives

- Calculate the mean and standard deviation of a sampling distribution of sample proportions
- Verify the two rules of thumb (conditions) before using the normal approximation for sample proportions
- Use the sampling distribution of sample proportions to find probabilities using the standard normal distribution

Problems

1. A population proportion is p = 0.40. What is the mean of the sampling distribution of the sample proportion?

$$\mu_{\hat{p}} = p$$

2. A population has proportion p = 0.60 and a sample of size n = 50 is drawn. Calculate the standard deviation of the sampling distribution of the sample proportion.

$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}$$

3. A factory has 2,000 workers and a sample of size n = 40 is selected. Check whether Rule of Thumb 1 is satisfied: the population must be at least 10 times the sample size.

$$N \geq 10n$$

4. A sample of size n = 25 is drawn from a population with proportion p = 0.30. Check whether Rule of Thumb 2 is satisfied: both np and nq must be greater than or equal to 10.

$$np \geq 10 \quad \text{and} \quad nq \geq 10$$

5. A sample of size n = 100 is drawn from a population with proportion p = 0.55 and population size N = 5,000. Verify both rules of thumb and state whether normal approximation may be used.

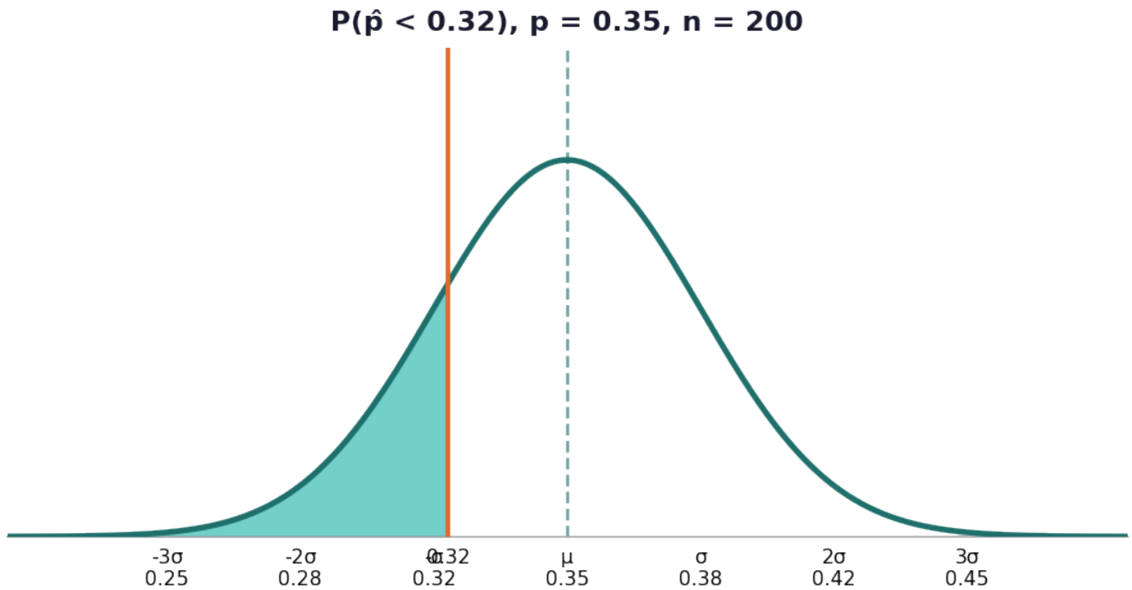
$$N \geq 10n \quad ; \quad np \geq 10 \quad ; \quad nq \geq 10$$

Scan to watch

6. In a large university, 45% of students own a laptop. A random sample of 80 students is selected. Find the mean and standard deviation of the sampling distribution of the sample proportion.

$$\mu_{\hat{p}} = p \quad , \quad \sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}$$

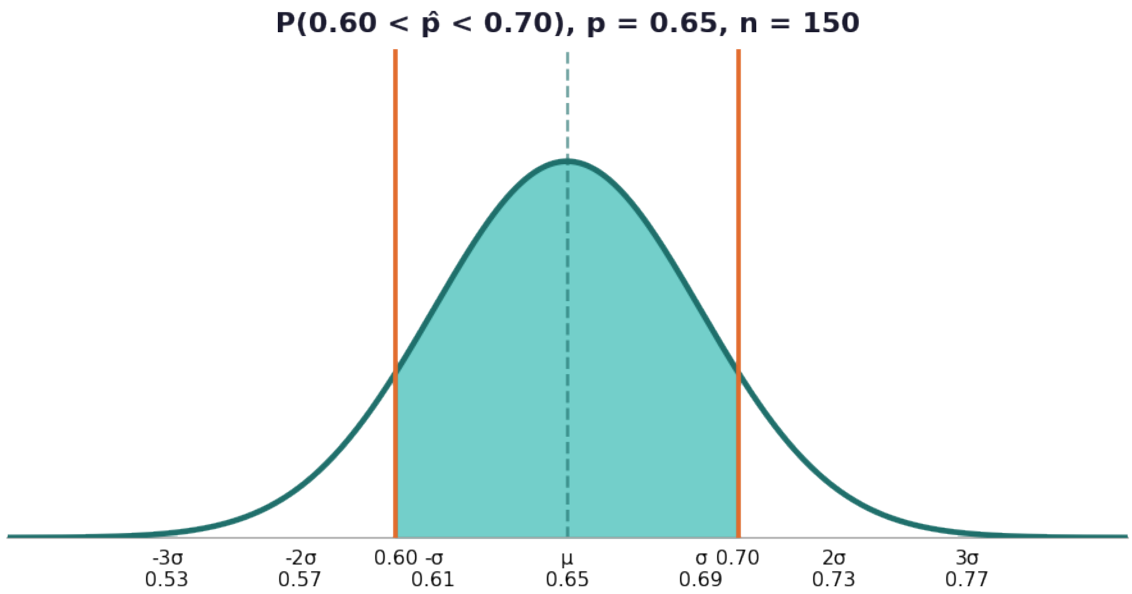
7. It is known that 35% of adults in a city exercise regularly. A random sample of 200 adults is taken from a city population of 50,000. After verifying both rules of thumb, find the probability that the sample proportion of adults who exercise is less than 0.32.



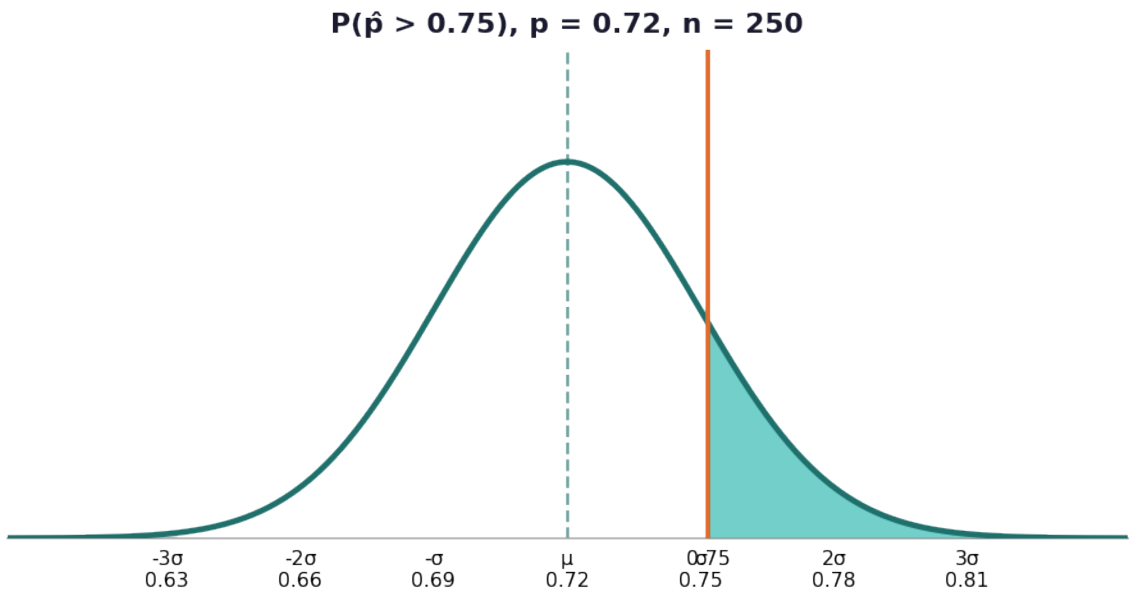
8. A factory employs 3,000 unionized workers, of whom 30% are Hispanic. A 15-member union executive committee is chosen at random. Check both rules of thumb and explain whether the normal approximation can be used to find the probability of three or fewer Hispanics on the committee.

$$np = 15 \times 0.30 = 4.5$$

9. A national survey reports that 65% of teenagers use social media daily. A researcher takes a random sample of 150 teenagers from a school district of 4,000. Find the probability that the sample proportion of daily social media users is between 0.60 and 0.70.



10. A pharmaceutical company claims that 72% of patients who take a new medication experience improvement. A hospital randomly selects 250 patients from a pool of 10,000 who received the medication. After verifying both rules of thumb, find the probability that more than 75% of the sampled patients show improvement. Then interpret what this probability means in context.



Scan to watch

Sampling Distribution of Sample Proportions — Answer Key

Statistics Worksheet · Grade 11–12 / Introductory College Statistics

Answer Key

1. Answer: $\mu_{\hat{p}} = 0.40$

- The mean of the sampling distribution of the sample proportion equals the population proportion.
- $\mu_{\hat{p}} = p = 0.40$

2. Answer: $\sigma_{\hat{p}} \approx 0.0693$

- Identify $q = 1 - p = 1 - 0.60 = 0.40$.
- Apply the formula: $\sigma_{\hat{p}} = \sqrt{(pq/n)} = \sqrt{(0.60 \times 0.40 / 50)} = \sqrt{(0.24/50)} = \sqrt{0.0048} \approx 0.0693$.

3. Answer: Yes, satisfied: 2,000 ≥ 400

- Calculate $10 \times n = 10 \times 40 = 400$.
- Compare: $N = 2{,}000 \geq 400$. Rule of Thumb 1 is satisfied.

4. Answer: Not satisfied: np = 7.5 < 10

- Calculate $np = 25 \times 0.30 = 7.5$. Since $7.5 < 10$, the first part of Rule of Thumb 2 fails.
- Calculate $nq = 25 \times 0.70 = 17.5 \geq 10$. The second part passes, but Rule of Thumb 2 is NOT satisfied overall because both conditions must hold.

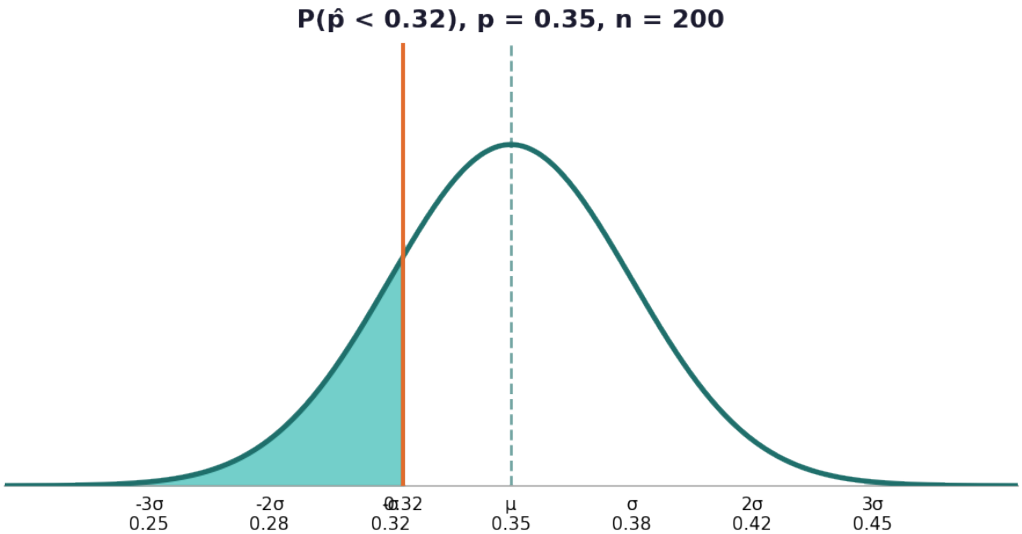
5. Answer: Both rules satisfied; normal approximation may be used.

- Rule 1: $10n = 10 \times 100 = 1{,}000$. $N = 5{,}000 \geq 1{,}000$. ✓
- Rule 2: $np = 100 \times 0.55 = 55 \geq 10$ ✓; $nq = 100 \times 0.45 = 45 \geq 10$ ✓. Both rules are satisfied, so the normal approximation is valid.

6. Answer: $\mu_{\hat{p}} = 0.45$; $\sigma_{\hat{p}} \approx 0.0556$

- Mean: $\mu_{\hat{p}} = p = 0.45$.
- Standard deviation: $q = 1 - 0.45 = 0.55$; $\sigma_{\hat{p}} = \sqrt{(0.45 \times 0.55 / 80)} = \sqrt{(0.2475/80)} = \sqrt{0.0030938} \approx 0.0556$.

7. Answer: $P(\hat{p} < 0.32) \approx 0.1867$ (18.67%)



- Verify Rule 1: 10 × 200 = 2,000 ≤ 50,000 ✓.
- Verify Rule 2: np = 200 × 0.35 = 70 ≥ 10 ✓; nq = 200 × 0.65 = 130 ≥ 10 ✓.
- Compute σ_p■ = √(0.35 × 0.65 / 200) = √(0.2275/200) = √0.0011375 ≈ 0.03372.
- Compute z = (0.32 – 0.35) / 0.03372 = –0.03 / 0.03372 ≈ –0.89.
- P(z < –0.89) ≈ 0.1867.

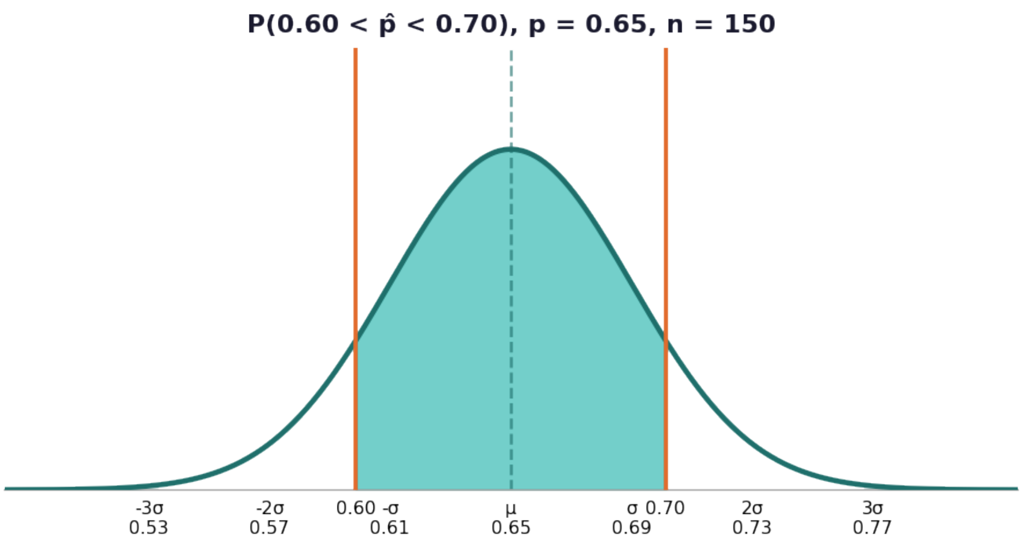
---

8. Answer: Cannot use normal approximation; Rule of Thumb 2 is not satisfied (np = 4.5 < 10).

- Rule 1: 10n = 10 × 15 = 150. N = 3,000 ≥ 150 ✓.
- Rule 2: np = 15 × 0.30 = 4.5 < 10 ✗. Rule of Thumb 2 is NOT satisfied.
- Since np < 10, the normal approximation is unreliable. The binomial distribution should be used instead.

---

9. Answer: P(0.60 < p■ < 0.70) ≈ 0.7994 (79.94%)



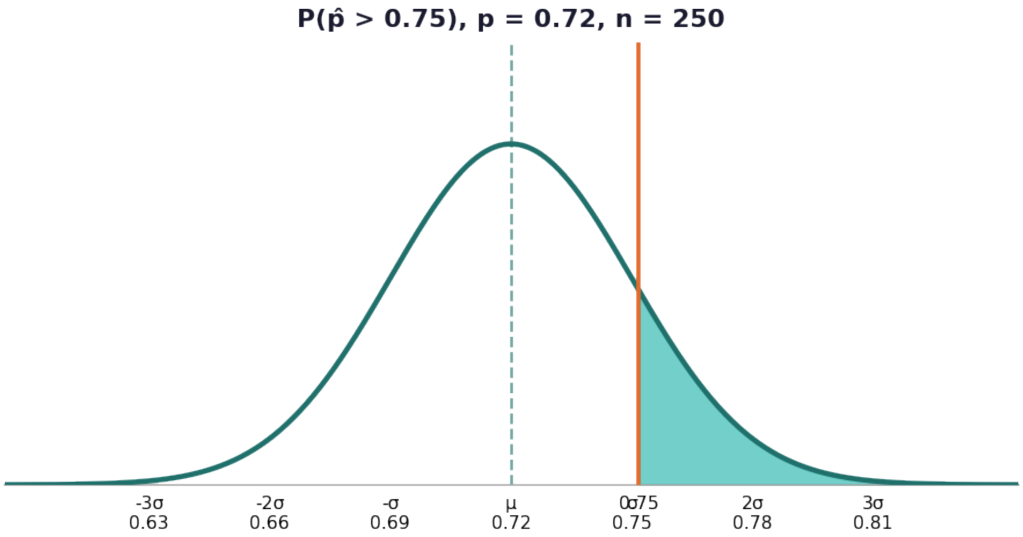
- Verify Rule 1: 10 × 150 = 1,500 ≤ 4,000 ✓.
- Verify Rule 2: np = 150 × 0.65 = 97.5 ≥ 10 ✓; nq = 150 × 0.35 = 52.5 ≥ 10 ✓.
- Compute σ_p■ = √(0.65 × 0.35 / 150) = √(0.2275/150) ≈ √0.001517 ≈ 0.03893.

Scan to watch

- z■ = (0.60 – 0.65) / 0.03893 ≈ –1.28; z■ = (0.70 – 0.65) / 0.03893 ≈ 1.28.
- P(–1.28 < z < 1.28) = P(z < 1.28) – P(z < –1.28) ≈ 0.8997 – 0.1003 = 0.7994.

---

10. Answer: P(p■ > 0.75) ≈ 0.1446 (14.46%); there is about a 14.46% chance that more than 75% of the sample shows improvement if the true rate is 72%.



- Verify Rule 1: 10 × 250 = 2,500 ≤ 10,000 ✓.
- Verify Rule 2: np = 250 × 0.72 = 180 ≥ 10 ✓; nq = 250 × 0.28 = 70 ≥ 10 ✓. Both rules satisfied.
- Compute σ_p■ = √(0.72 × 0.28 / 250) = √(0.2016/250) = √0.0008064 ≈ 0.02838.
- Compute z = (0.75 – 0.72) / 0.02838 = 0.03 / 0.02838 ≈ 1.06.
- P(z > 1.06) = 1 – P(z < 1.06) ≈ 1 – 0.8554 = 0.1446.
- Interpretation: If the true improvement rate is 72%, there is approximately a 14.46% probability of observing a sample proportion above 75% purely by chance. This is not unusually high, so the company's claim is plausible.

Scan to watch