Sampling Distributions: Mean & Standard Deviation

Statistics Worksheet · Grade 11–12 / Introductory College

Name: ________________________________ Date: ____________________

Learning Objectives

- Apply the Central Limit Theorem to identify the mean of a sampling distribution
- Calculate the standard deviation (standard error) of a sampling distribution using the formula σ/√n
- Solve probability questions involving sampling distributions using z-scores and the normal curve

Problems

1. A population has a mean of 50. According to the Central Limit Theorem, what is the mean of the sampling distribution of sample means drawn from this population?

$$\mu_{\bar{x}} = ?$$

2. A population has a standard deviation of 12. A random sample of size 36 is drawn. Find the standard deviation of the sampling distribution (standard error).

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{36}}$$

3. The heights of young women are normally distributed with a population mean of 64.5 inches and a population standard deviation of 2.5 inches. If 10 women are randomly selected, what is the mean of the sampling distribution?

$$\mu_{\bar{x}} = ?$$

4. Using the same women's height distribution (mean = 64.5 inches, standard deviation = 2.5 inches), find the standard deviation of the sampling distribution when 10 women are randomly selected. Round to two decimal places.

$$\sigma_{\bar{x}} = \frac{2.5}{\sqrt{10}}$$

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5. Using the women's height distribution (mean = 64.5 inches, standard deviation = 2.5 inches), compute the standard deviation of the sampling distribution when the sample size increases to 100. How does this compare to the result for n = 10?

$$\sigma_{\bar{x}} = \frac{2.5}{\sqrt{100}}$$

6. Fill in the missing values in the table below for the women's height distribution (μ = 64.5, σ = 2.5). Round standard errors to two decimal places.

Sample Size (n)	Sample Mean (μ_x̄)	Standard Error (σ_x̄)
10	64.5	
25	64.5	
100	64.5	
400	64.5	

7. A manufacturer fills juice bottles with a mean of 16 oz and a standard deviation of 0.8 oz. A quality inspector randomly selects 64 bottles. Find the mean and standard deviation of the sampling distribution of the sample mean.

$$\mu_{\bar{x}} = \mu, \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

8. The heights of young women are normally distributed with a mean of 64.5 inches and a standard deviation of 2.5 inches. A sample of 10 women is randomly selected. Find the probability that the sample mean height is greater than 65.5 inches. Use the standard normal distribution.

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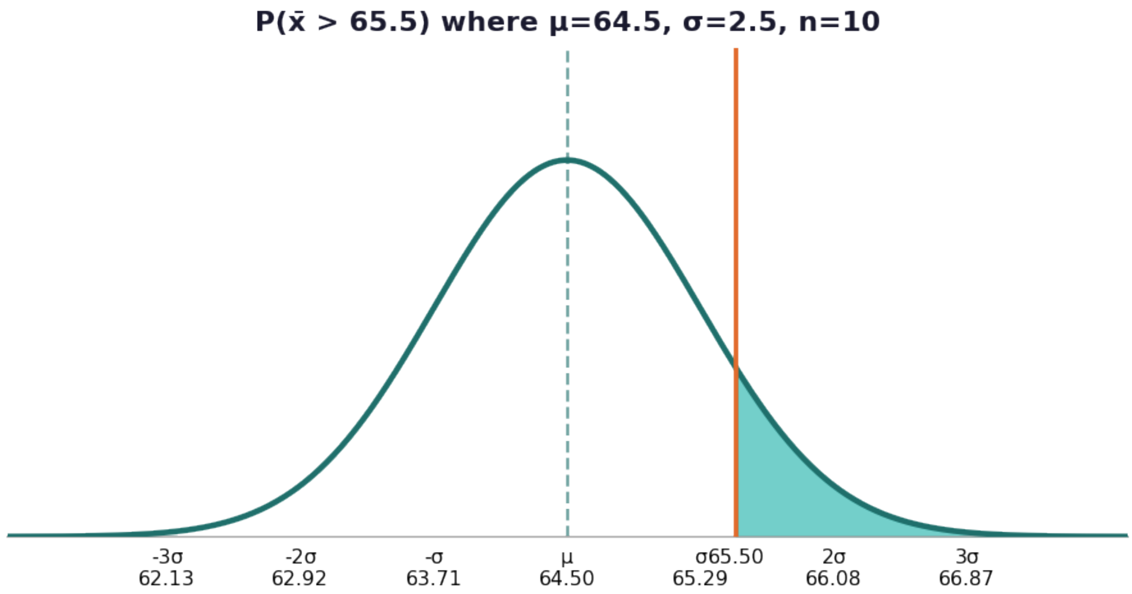
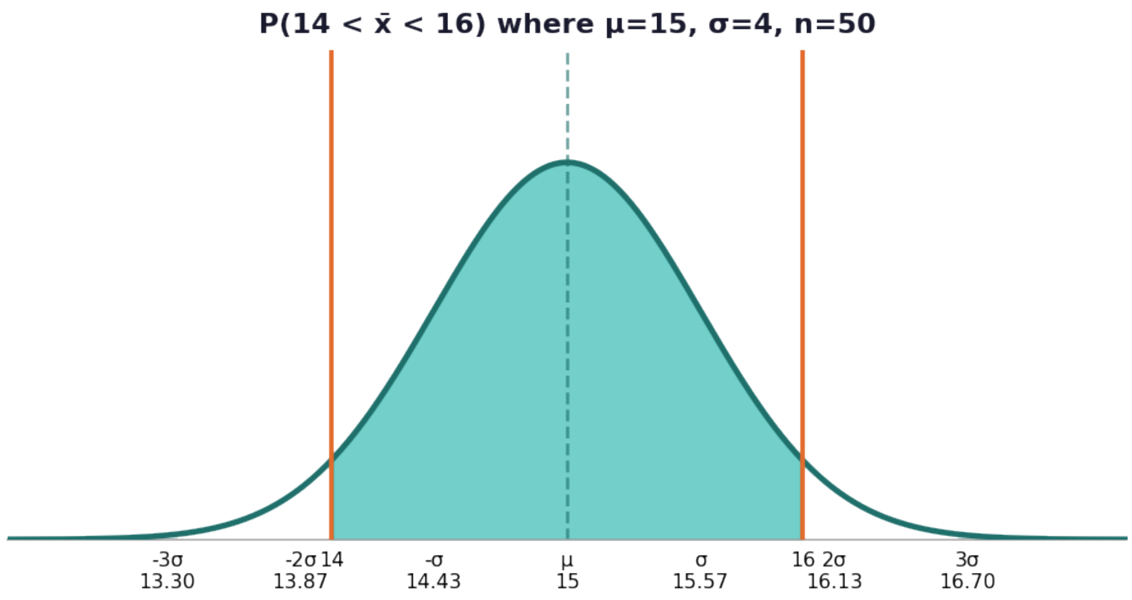


9. A university reports that the average student studies 15 hours per week with a standard deviation of 4 hours. A random sample of 50 students is taken. What is the probability that the sample mean study time is between 14 and 16 hours per week?



10. Scores on a standardized exam are normally distributed with a mean of 500 and a standard deviation of 100. A researcher draws a random sample of 25 students. (a) Find the mean and standard error of the sampling distribution. (b) Find the probability that the sample mean score is less than 480. (c) A second

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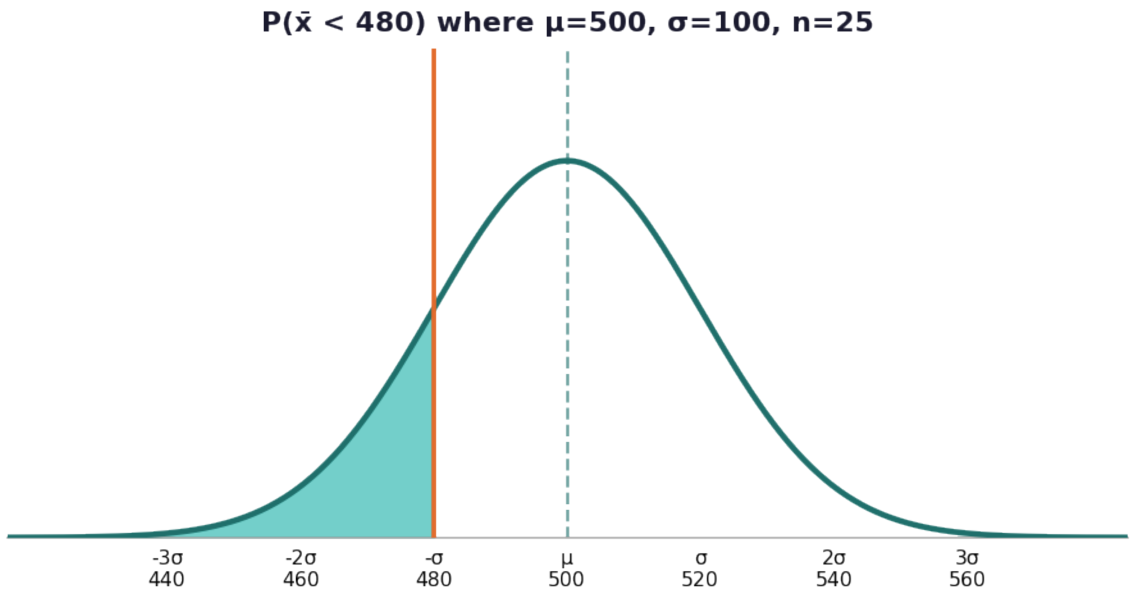
researcher uses a sample of 100 students instead. Without computing, explain how this changes the standard error and the probability in part (b).



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Sampling Distributions: Mean & Standard Deviation — Answer Key

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Answer Key

1. Answer: μ_x■ = 50

- The Central Limit Theorem states that the mean of the sampling distribution equals the population mean.
- Therefore μ_x■ = μ = 50.

2. Answer: σ_x■ = 2

- Use the formula σ_x■ = σ / √n.
- σ_x■ = 12 / √36 = 12 / 6 = 2.

3. Answer: μ_x■ = 64.5 inches

- By the Central Limit Theorem, the sampling distribution mean equals the population mean.
- Therefore μ_x■ = 64.5 inches.

4. Answer: σ_x■ ≈ 0.79 inches

- Use σ_x■ = σ / √n = 2.5 / √10.
- √10 ≈ 3.162, so σ_x■ = 2.5 / 3.162 ≈ 0.79 inches.

5. Answer: σ_x■ = 0.25 inches; smaller than the n=10 result of 0.79 inches

- σ_x■ = 2.5 / √100 = 2.5 / 10 = 0.25 inches.
- For n = 10, σ_x■ ≈ 0.79 inches. As sample size increases, the standard error decreases.

6. Answer: See completed table

Sample Size (n)	Sample Mean (μ_x̄)	Standard Error (σ_x̄)
10	64.5	0.79
25	64.5	0.50
100	64.5	0.25
400	64.5	0.13

- For each row, apply σ_x■ = 2.5 / √n.
- n=10: 2.5/√10 ≈ 0.79 | n=25: 2.5/√25 = 0.50 | n=100: 2.5/√100 = 0.25 | n=400: 2.5/√400 = 0.13.

7. Answer: μ_x■ = 16 oz, σ_x■ = 0.1 oz

- Mean of sampling distribution: μ_x■ = μ = 16 oz.
- Standard error: σ_x■ = 0.8 / √64 = 0.8 / 8 = 0.1 oz.

8. Answer: P ≈ 0.1027 (10.27%)

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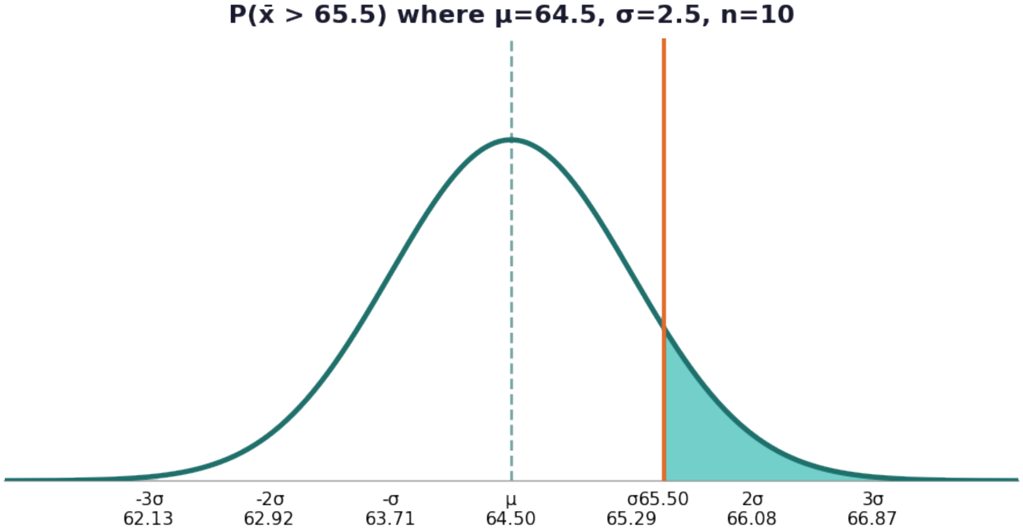

- Step 1: Find σ_x■ = 2.5 / √10 ≈ 0.79 inches.
- Step 2: Compute z = (65.5 – 64.5) / 0.79 = 1.0 / 0.79 ≈ 1.27.
- Step 3: P(z > 1.27) = 1 – P(z < 1.27) = 1 – 0.8973 ≈ 0.1027.
- The probability that the sample mean exceeds 65.5 inches is approximately 10.27%.

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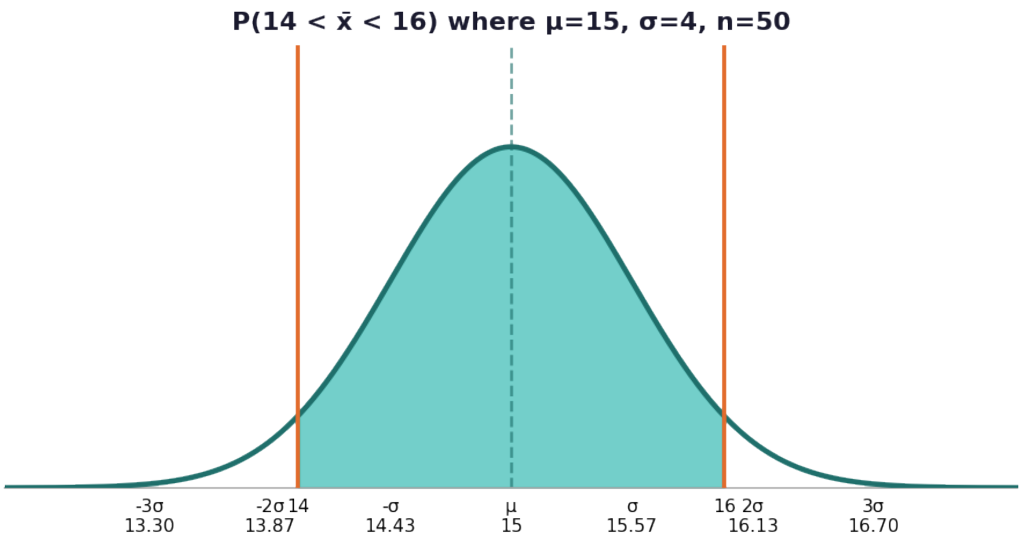
9. Answer: P ≈ 0.9216 (92.16%)



- Step 1: σ_x■ = 4 / √50 ≈ 0.566 hours.
- Step 2: z■ = (14 – 15) / 0.566 ≈ –1.77 and z■ = (16 – 15) / 0.566 ≈ 1.77.
- Step 3: P(–1.77 < z < 1.77) = P(z < 1.77) – P(z < –1.77) = 0.9616 – 0.0384 ≈ 0.9232.
- The probability that the sample mean is between 14 and 16 hours is approximately 92.32%.

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10. Answer: (a) μ_x■ = 500, σ_x■ = 20; (b) P ≈ 0.1587 (15.87%); (c) σ_x■ drops to 10, making it less likely to get x■ < 480

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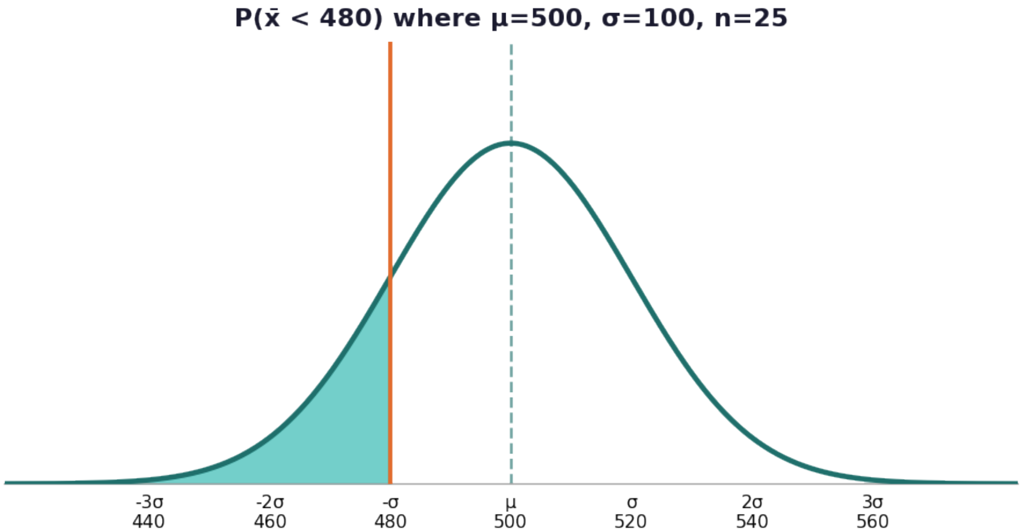

- (a) μ_x■ = 500 (equals population mean); σ_x■ = 100 / √25 = 100 / 5 = 20.
- (b) z = (480 – 500) / 20 = –20 / 20 = –1.00.
- P(z < –1.00) = 0.1587, so the probability is approximately 15.87%.
- (c) With n = 100, σ_x■ = 100 / √100 = 10. The sampling distribution is narrower.
- The z-score becomes (480 – 500) / 10 = –2.00, giving P ≈ 0.0228 — much smaller probability.

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