

# Sampling Distributions & Normal Probability

Statistics Worksheet · Grade 11–12 / Introductory College

Name: \_\_\_\_\_

Date: \_\_\_\_\_

## Learning Objectives

- Calculate the mean and standard deviation of a sample proportion distribution
- Identify the range of values within a given percentage point margin of a true proportion
- Use z-scores and the normal CDF to find probabilities for sample proportions

## Problems

1. A survey finds that 40% of adults prefer coffee over tea. If you take a random sample of 100 adults, what is the mean of the sampling distribution of the sample proportion?

$$\mu_{\hat{p}} = p$$

2. A true population proportion is  $p = 0.60$  and a sample of size  $n = 200$  is taken. Calculate the standard deviation of the sampling distribution of the sample proportion.

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

3. The true proportion of college freshmen who own a laptop is 0.35. A sample of 1,500 freshmen is taken. What are the mean and standard deviation of the sampling distribution of the sample proportion?

$$\mu_{\hat{p}} = p, \quad \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

4. Suppose the true proportion is 0.35. What range of sample proportions falls within 2 percentage points of the true value? Express the range as a lower bound and an upper bound.

$$p \pm 0.02$$

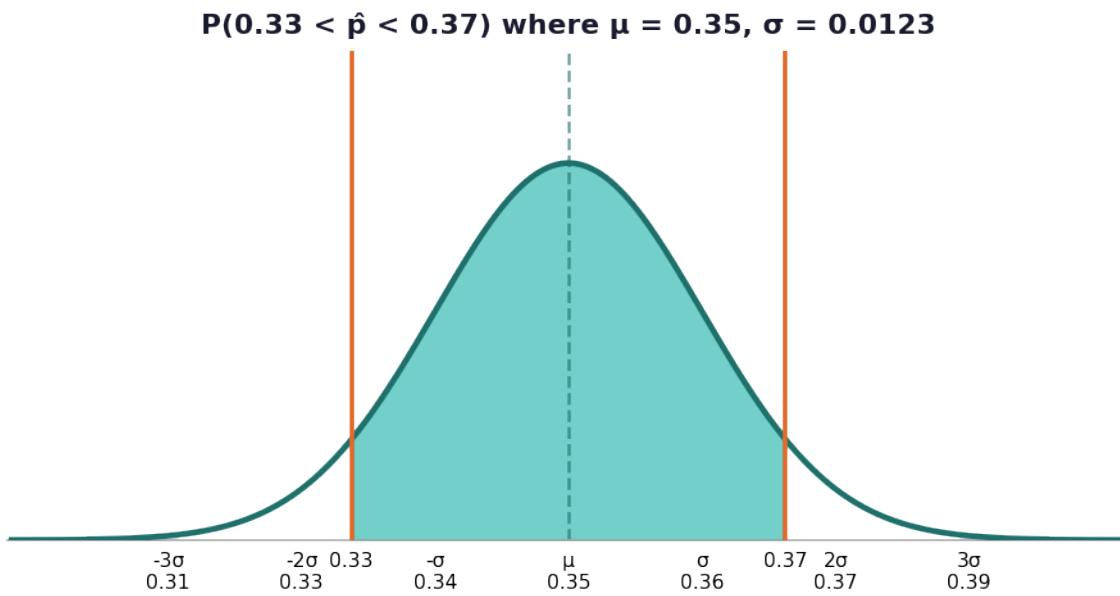
5. Using the normal distribution with mean 0.35 and standard deviation 0.0123, find the z-scores that correspond to a sample proportion of 0.33 and a sample proportion of 0.37.

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$$Z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}}$$

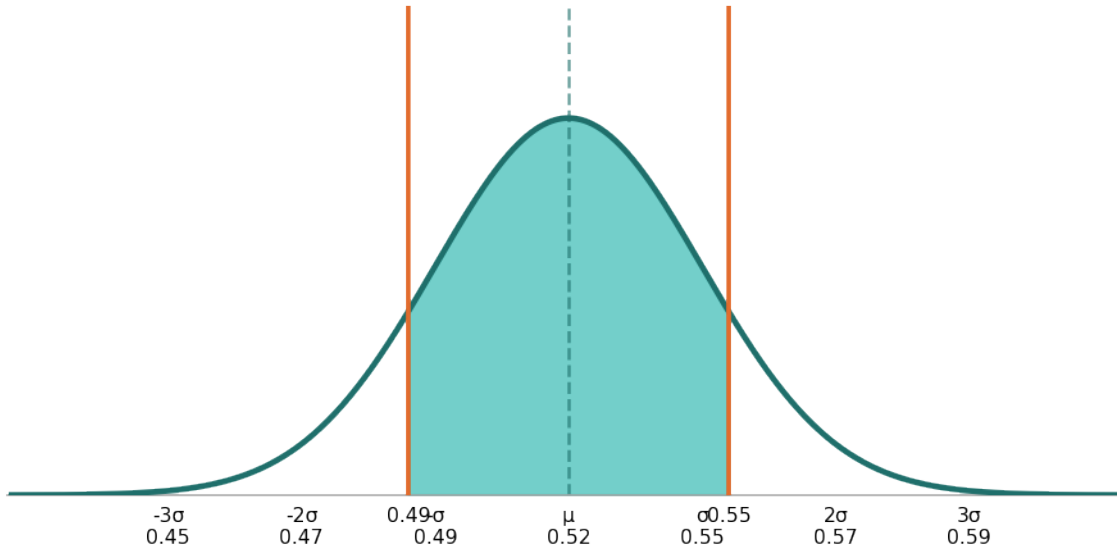
6. Based on the sampling distribution with mean 0.35 and standard deviation 0.0123, use the normal curve below to visually identify the shaded probability region between sample proportions 0.33 and 0.37. Then find the probability.



7. A poll shows that 52% of voters support a new policy. A random sample of 400 voters is taken. What is the probability that the sample proportion is within 3 percentage points of the true proportion of 0.52?



**$P(0.49 < \hat{p} < 0.55)$  where  $\mu = 0.52, \sigma \approx 0.025$**



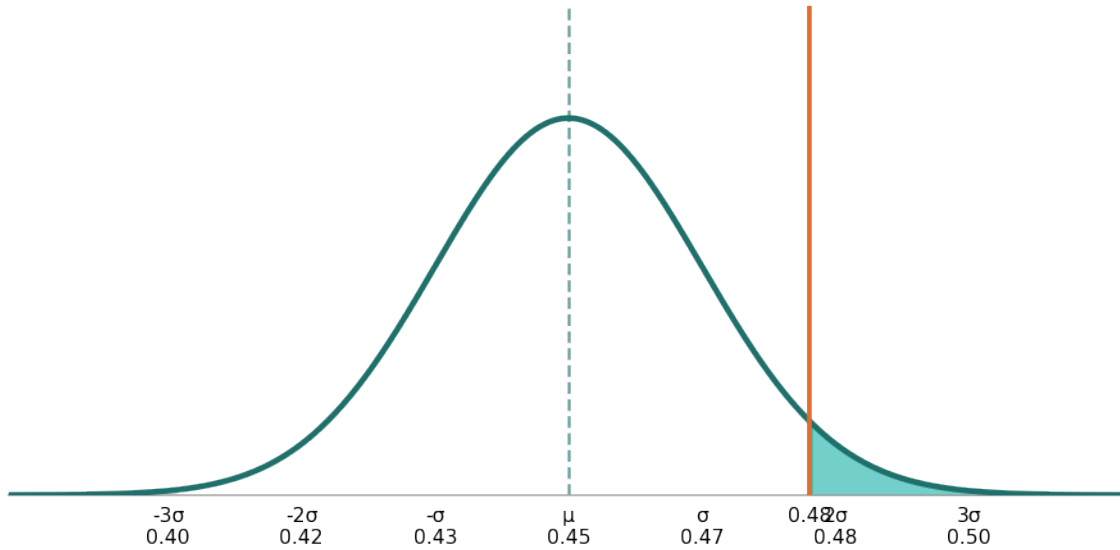
8. The table below shows sample sizes and true proportions for four different surveys. Complete the missing standard deviations of the sampling distribution of the sample proportion.

Survey	p (true proportion)	n (sample size)	$\sigma_{\hat{p}}$
A	0.35	1500	0.0123
B	0.50	1000	
C	0.20	500	
D	0.75	800	

9. A true population proportion is 0.45. A sample of 900 is taken. What is the probability that the sample proportion will be greater than 0.48? Round to four decimal places.

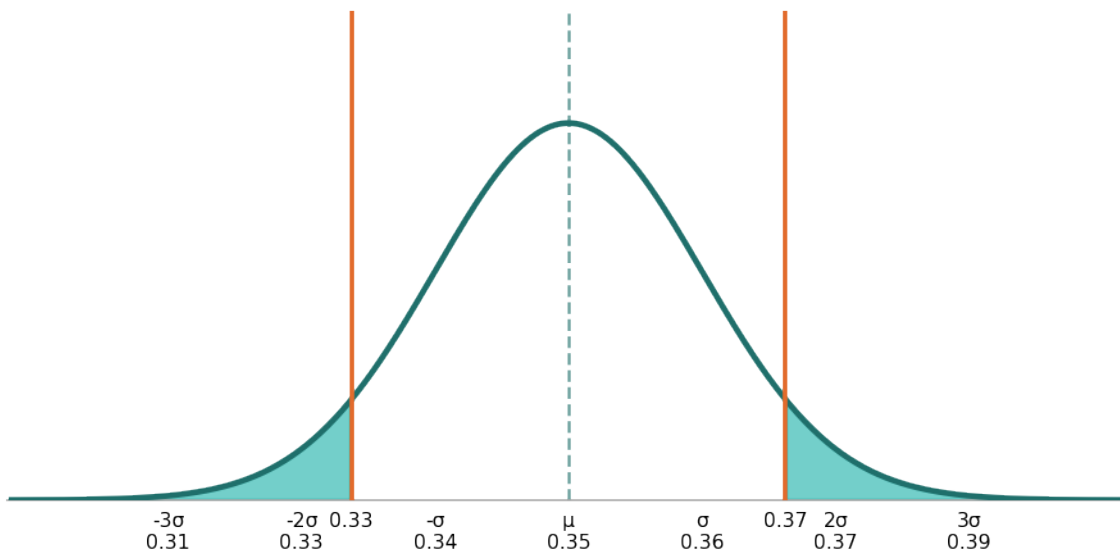


**$P(\hat{p} > 0.48)$  where  $\mu = 0.45, \sigma \approx 0.0166$**



**10.** A researcher claims that 35% of high school seniors plan to major in STEM. In a study of 1,500 students, what is the probability that the sample proportion falls OUTSIDE the range of 0.33 to 0.37 (i.e., more than 2 percentage points away from the true proportion)? Use the sampling distribution with mean 0.35 and standard deviation 0.0123.

**$P(\hat{p} < 0.33 \text{ or } \hat{p} > 0.37)$  where  $\mu = 0.35, \sigma = 0.0123$**



# Sampling Distributions & Normal Probability — Answer Key

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## Answer Key

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### 1. Answer: 0.40

- The mean of the sampling distribution of the sample proportion equals the true population proportion.
- $\mu_{\hat{p}} = p = 0.40$

### 2. Answer: $\approx 0.0346$

- Identify  $p = 0.60$ ,  $1 - p = 0.40$ ,  $n = 200$ .
- $\sigma_{\hat{p}} = \sqrt{(0.60 \times 0.40 / 200)} = \sqrt{(0.24 / 200)} = \sqrt{0.0012} \approx 0.0346$

### 3. Answer: Mean = 0.35, Standard Deviation $\approx 0.0123$

- $\mu_{\hat{p}} = p = 0.35$
- $\sigma_{\hat{p}} = \sqrt{(0.35 \times 0.65 / 1500)} = \sqrt{(0.2275 / 1500)} = \sqrt{0.0001517} \approx 0.0123$

### 4. Answer: 0.33 to 0.37

- Lower bound:  $0.35 - 0.02 = 0.33$
- Upper bound:  $0.35 + 0.02 = 0.37$
- The range is  $[0.33, 0.37]$ .

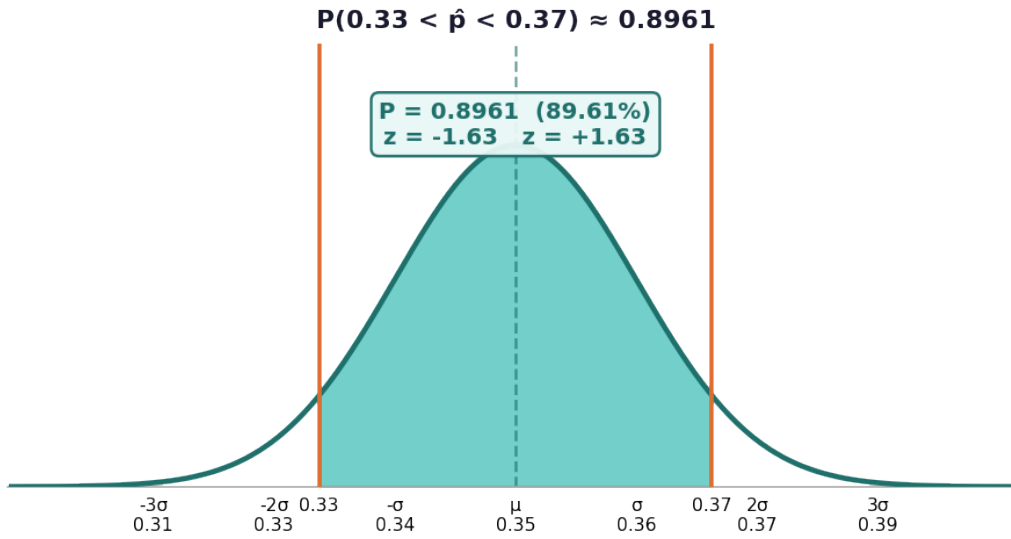
### 5. Answer: $z = -1.63$ and $z = 1.63$

- For 0.33:  $z = (0.33 - 0.35) / 0.0123 = -0.02 / 0.0123 \approx -1.63$
- For 0.37:  $z = (0.37 - 0.35) / 0.0123 = 0.02 / 0.0123 \approx 1.63$

### 6. Answer: $P \approx 0.8961$ (89.61%)

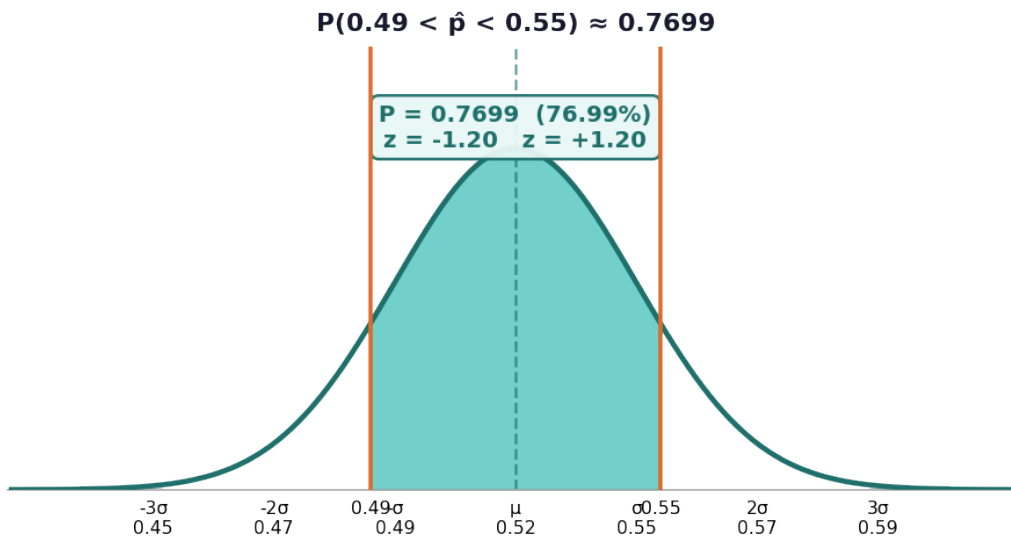
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- $z_{\text{left}} = (0.33 - 0.35) / 0.0123 \approx -1.63$
- $z_{\text{right}} = (0.37 - 0.35) / 0.0123 \approx 1.63$
- $P(-1.63 < z < 1.63) = \Phi(1.63) - \Phi(-1.63) \approx 0.9484 - 0.0516 = 0.8968 \approx 0.8961$

**7. Answer:  $P \approx 0.7699$  (76.99%)**



- $\sigma_{\hat{p}} = \sqrt{(0.52 \times 0.48 / 400)} = \sqrt{(0.2496 / 400)} = \sqrt{0.000624} \approx 0.025$
- Range within 3 pp:  $0.52 \pm 0.03 \rightarrow [0.49, 0.55]$
- $z_{\text{left}} = (0.49 - 0.52) / 0.025 = -1.20$ ,  $z_{\text{right}} = (0.55 - 0.52) / 0.025 = 1.20$
- $P(-1.20 < z < 1.20) \approx 0.8849 - 0.1151 = 0.7698 \approx 0.7699$

**8. Answer: B  $\approx 0.0158$ , C  $\approx 0.0179$ , D  $\approx 0.0153$**

Survey	p (true proportion)	n (sample size)	$\sigma_{\hat{p}}$
A	0.35	1500	0.0123

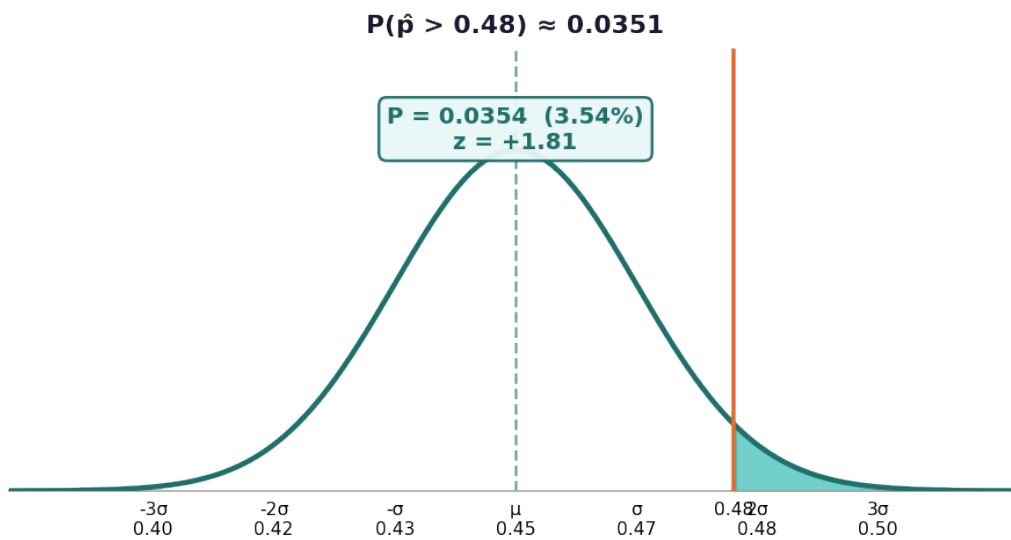
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Survey	p (true proportion)	n (sample size)	$\sigma_{\hat{p}}$
B	0.50	1000	0.0158
C	0.20	500	0.0179
D	0.75	800	0.0153

- B:  $\sqrt{(0.50 \times 0.50 / 1000)} = \sqrt{(0.25 / 1000)} = \sqrt{0.00025} \approx 0.0158$
- C:  $\sqrt{(0.20 \times 0.80 / 500)} = \sqrt{(0.16 / 500)} = \sqrt{0.00032} \approx 0.0179$
- D:  $\sqrt{(0.75 \times 0.25 / 800)} = \sqrt{(0.1875 / 800)} = \sqrt{0.000234} \approx 0.0153$

**9. Answer: P ≈ 0.0351 (3.51%)**

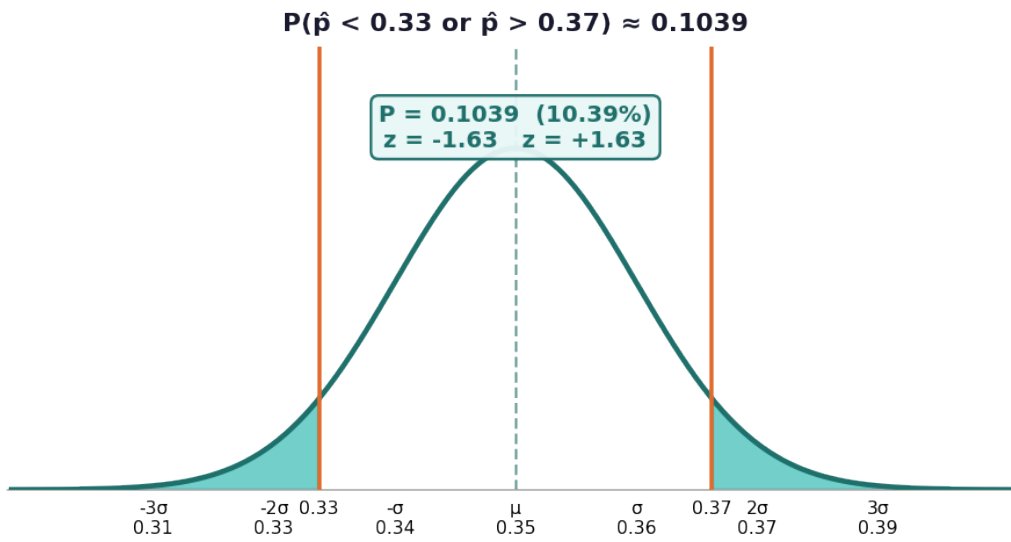


- $\sigma_{\hat{p}} = \sqrt{(0.45 \times 0.55 / 900)} = \sqrt{(0.2475 / 900)} = \sqrt{0.000275} \approx 0.0166$
- $z = (0.48 - 0.45) / 0.0166 = 0.03 / 0.0166 \approx 1.81$
- $P(z > 1.81) = 1 - \Phi(1.81) = 1 - 0.9649 = 0.0351$

**10. Answer: P ≈ 0.1039 (10.39%)**

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- From Problem 6,  $P(0.33 < \hat{p} < 0.37) \approx 0.8961$ .
- $P(\text{outside}) = 1 - P(\text{inside}) = 1 - 0.8961 = 0.1039$
- Alternatively:  $z = -1.63$  and  $z = 1.63$ .
- $P(z < -1.63) + P(z > 1.63) = 0.0516 + 0.0516 \approx 0.1039$  (about 10.39%)

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