Normal Distribution & Sampling Distribution Probabilities

Statistics Worksheet · Grade 11–12 / College Intro Stats

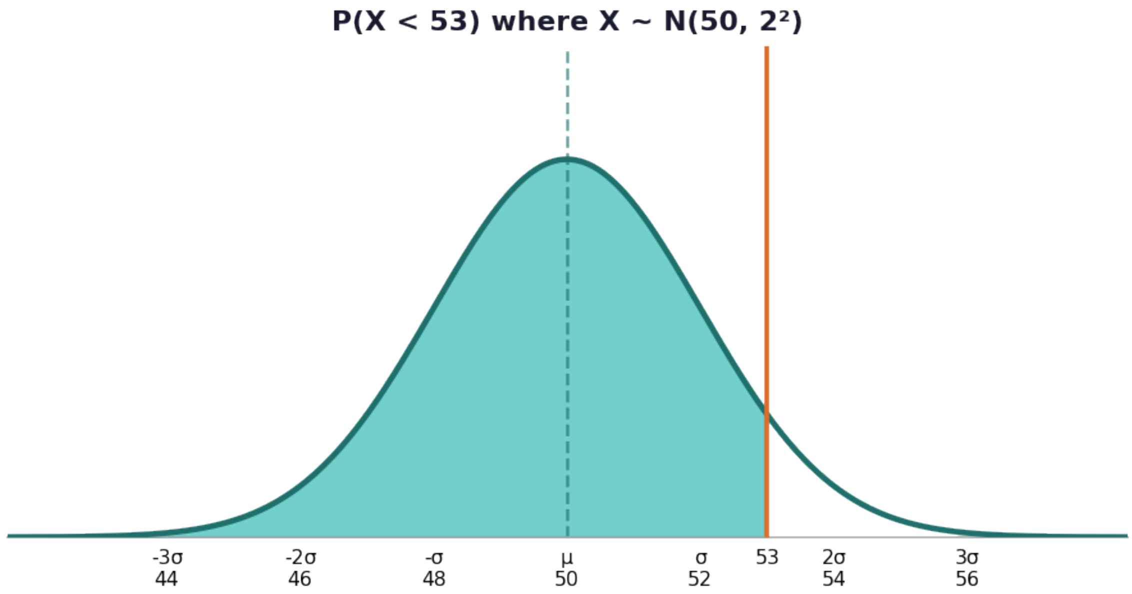
Name: ________________________________ Date: ___________________

Learning Objectives

- Use the TI-84 normalcdf function to compute probabilities under a normal distribution
- Apply the Central Limit Theorem to find probabilities for sample means using the standard error $\sigma/\sqrt{n}$
- Interpret probability results in the context of real-world word problems

Problems

1. A factory produces bolts whose lengths follow a normal distribution with a mean of 50 mm and a standard deviation of 2 mm. Using the TI-84 normalcdf function, find the probability that a randomly selected bolt is less than 53 mm long.



2. The heights of adult women in a city are normally distributed with a mean of 64 inches and a standard deviation of 3 inches. Using normalcdf on the TI-84, find the probability that a randomly selected woman is taller than 68 inches.

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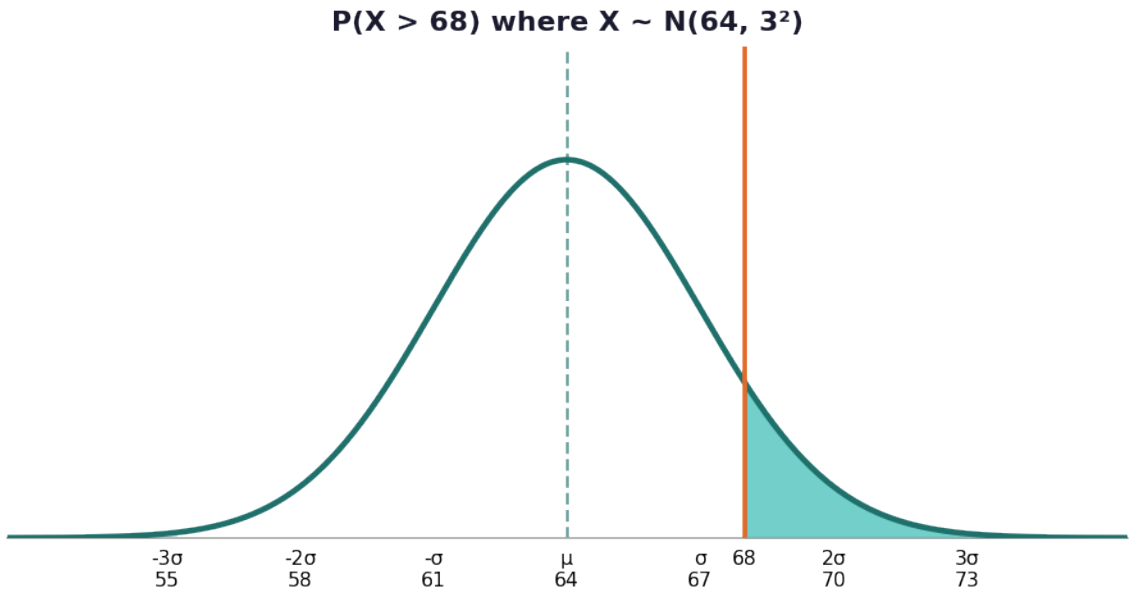
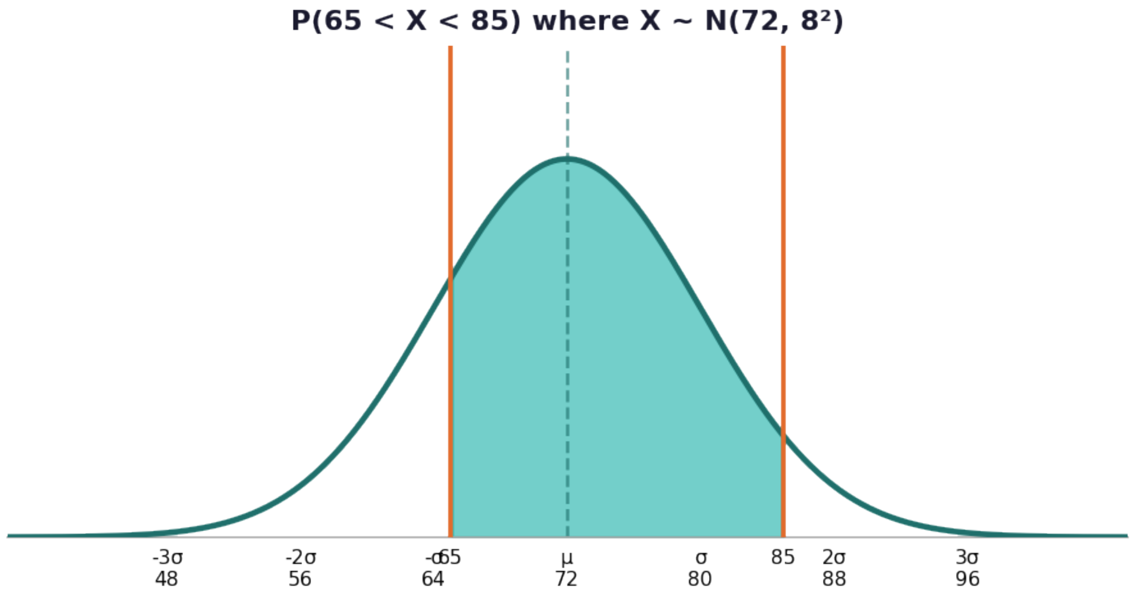


3. Exam scores at a university follow a normal distribution with a mean of 72 and a standard deviation of 8. Using the TI-84 normalcdf function, find the probability that a randomly selected student scored between 65 and 85.



4. The print on the package of 100-watt GE soft white light bulbs states that the bulbs have an average lifespan of 730 hours. Assume the lifespan follows a normal distribution with a mean of 730 hours and a standard deviation of 52 hours. For a random sample of 27 light bulbs, find the probability that the sample

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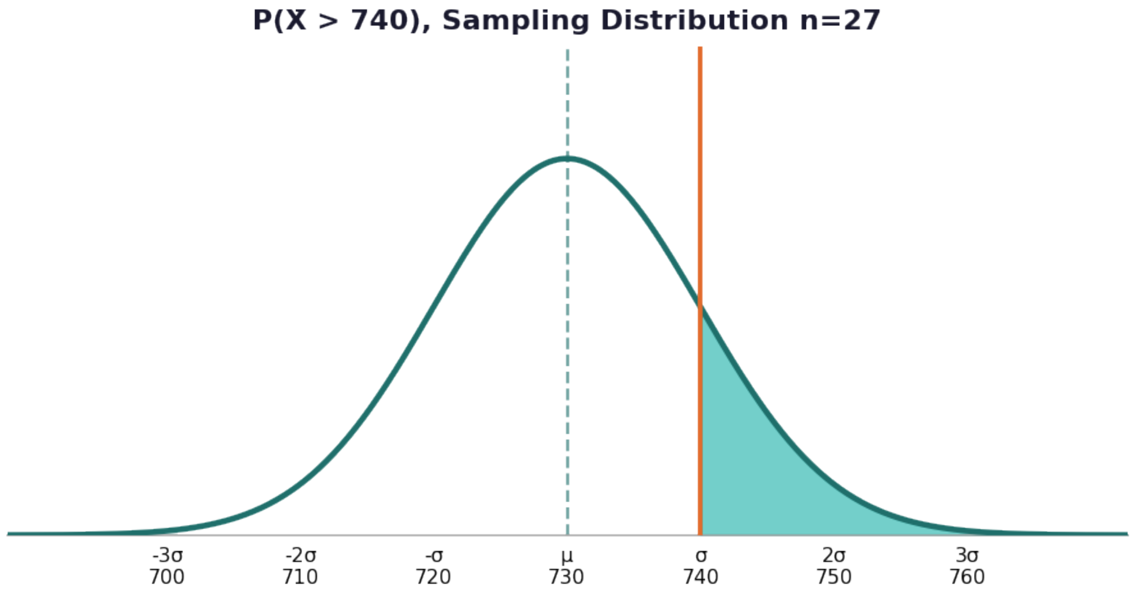
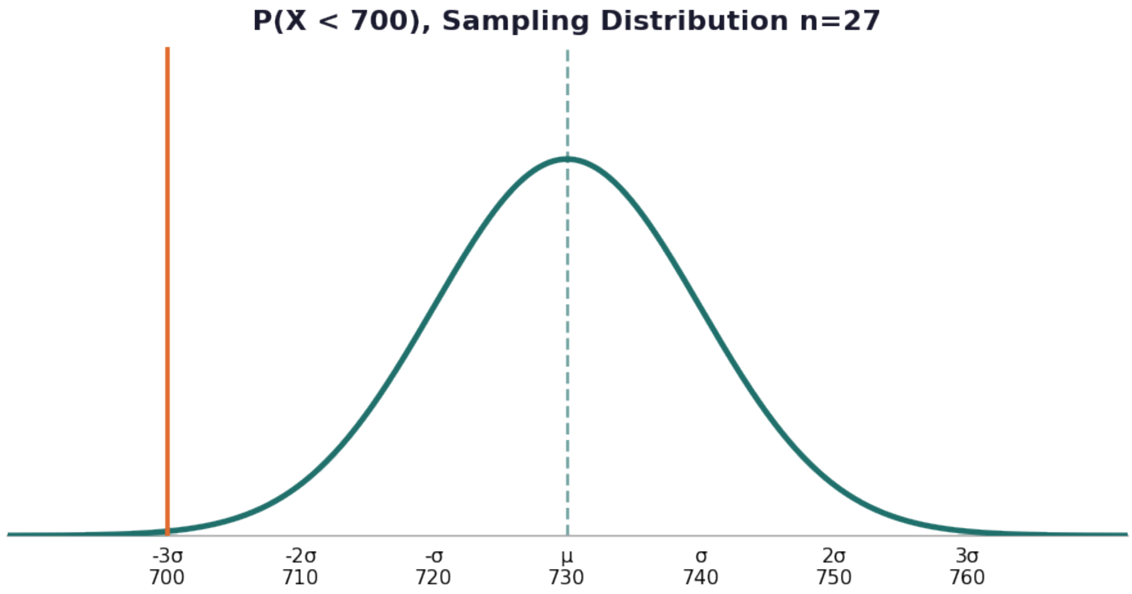
mean lifespan is greater than 740 hours.



5. Using the same light bulb scenario (mean = 730 hours, standard deviation = 52 hours, sample size = 27), find the probability that the sample mean lifespan is less than 700 hours.



6. Using the same light bulb scenario (mean = 730 hours, standard deviation = 52 hours, sample size = 27), find the probability that the sample mean lifespan is between 715 hours and 737 hours.

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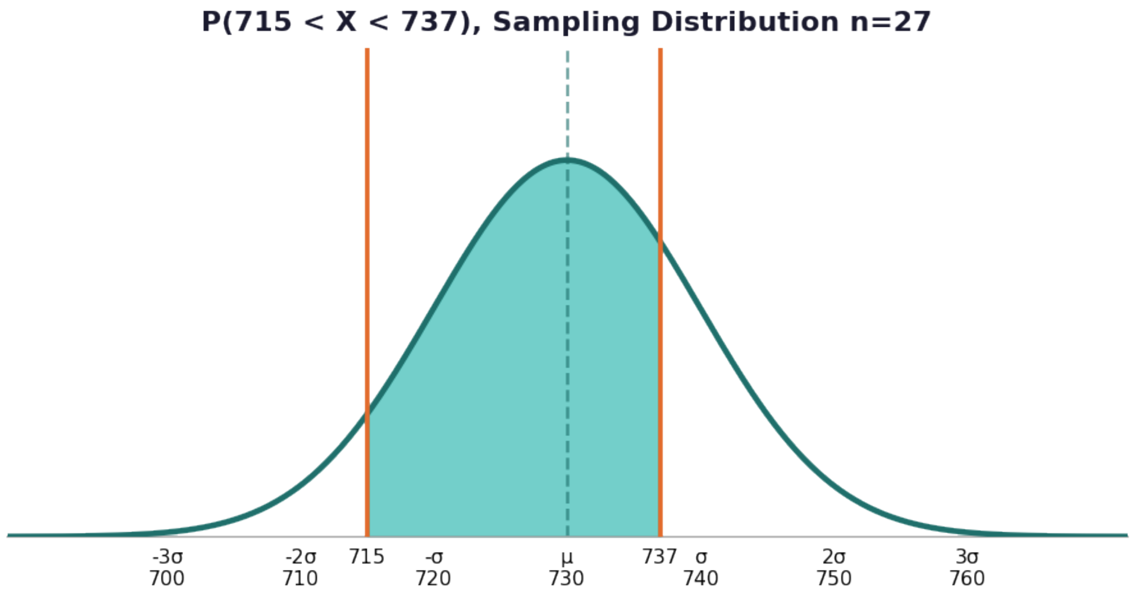
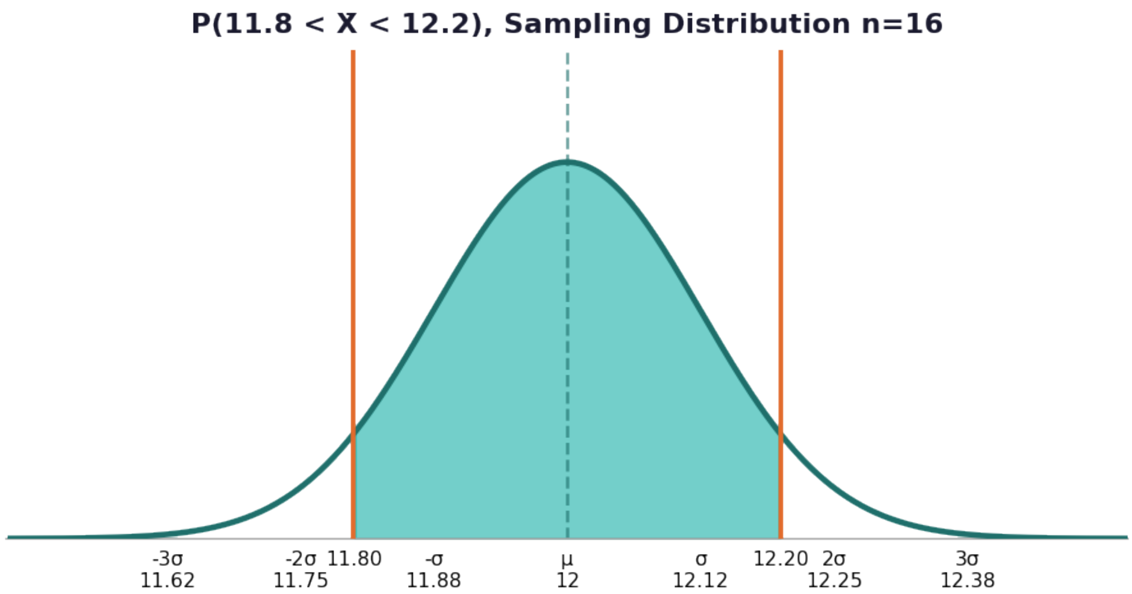


7. A coffee machine dispenses amounts that follow a normal distribution with a mean of 12 oz and a standard deviation of 0.5 oz. A random sample of 16 cups is selected. Find the probability that the sample mean amount dispensed is between 11.8 oz and 12.2 oz.



8. The weights of bags of flour filled by a machine follow a normal distribution with a mean of 5 lbs and a standard deviation of 0.08 lbs. A quality control inspector randomly samples 20 bags. Find the probability that the sample mean weight is less than 4.97 lbs OR greater than 5.03 lbs. (Hint: use the complement or

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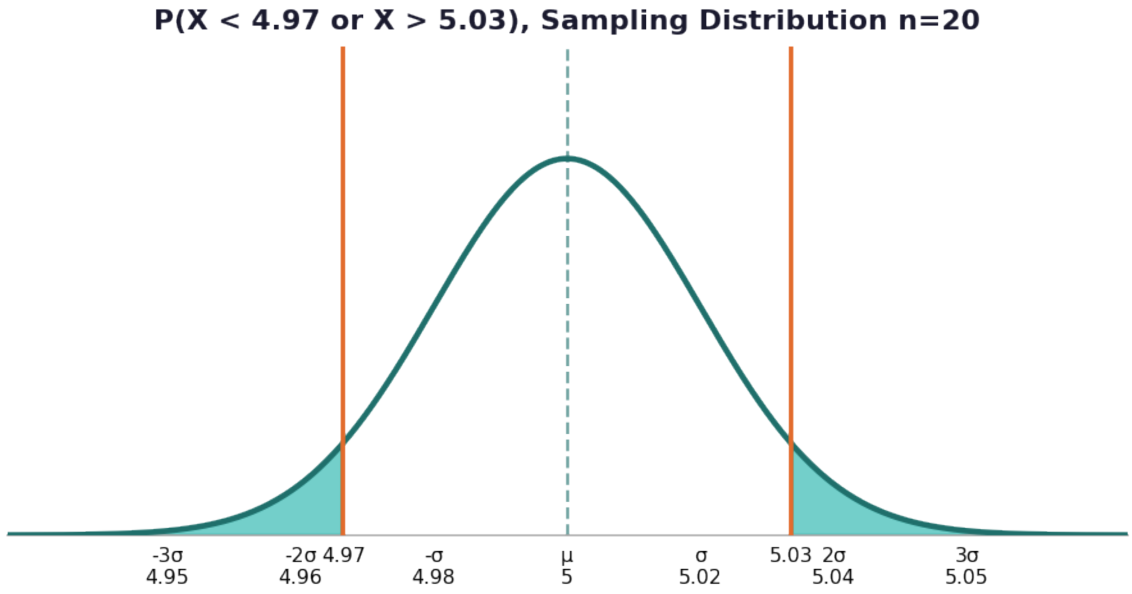
add both tail areas.)



9. The fill table below shows probability results from three different sampling scenarios for the light bulb problem. All use a normal distribution with mean 730 hours and standard deviation 52 hours. Using the TI-84, compute the missing probabilities and fill in the blank cells.

Scenario	Sample Size (n)	Condition	Probability
A	27	$P(\bar{X} > 740)$	
B	27	$P(\bar{X} < 700)$	
C	50	$P(\bar{X} > 740)$	
D	50	$P(\bar{X} < 700)$	
E	100	$P(\bar{X} > 740)$	

10. The annual salary of entry-level software engineers in a city follows a normal distribution with a mean of 72,000 dollars and a standard deviation of 9,500 dollars. A tech company randomly selects 35 recent hires. (a) Find the probability that the sample mean salary is greater than 75,000 dollars. (b) Find the probability that the sample mean salary is between 69,000 and 74,500 dollars. (c) Explain in one sentence how increasing the sample size from 35 to 100 would change the probability in part (a).

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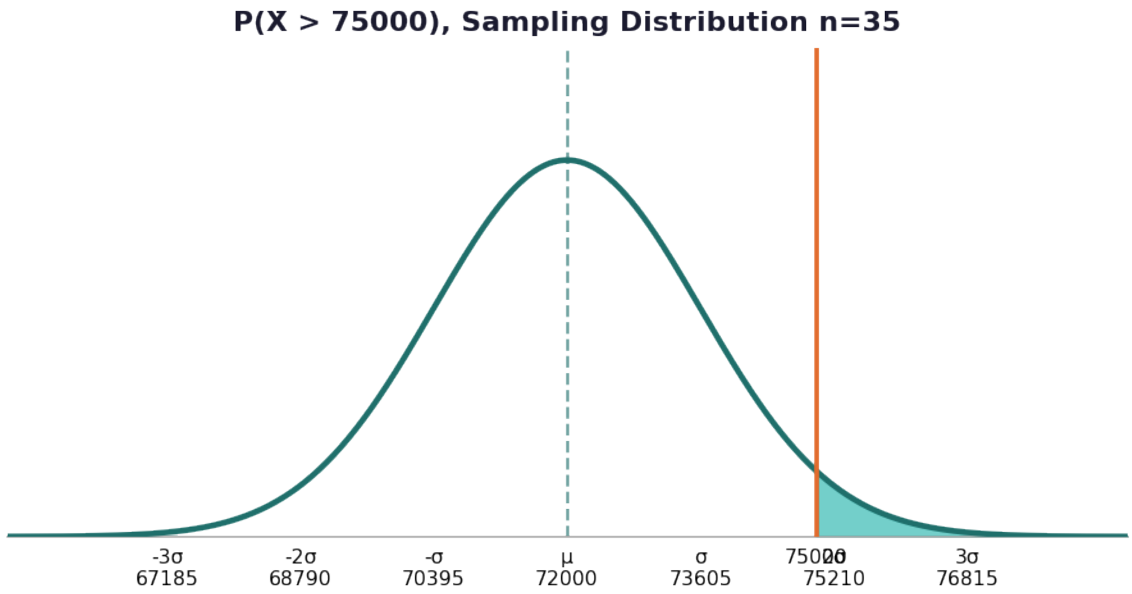

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Normal Distribution & Sampling Distribution Probabilities — Answer Key

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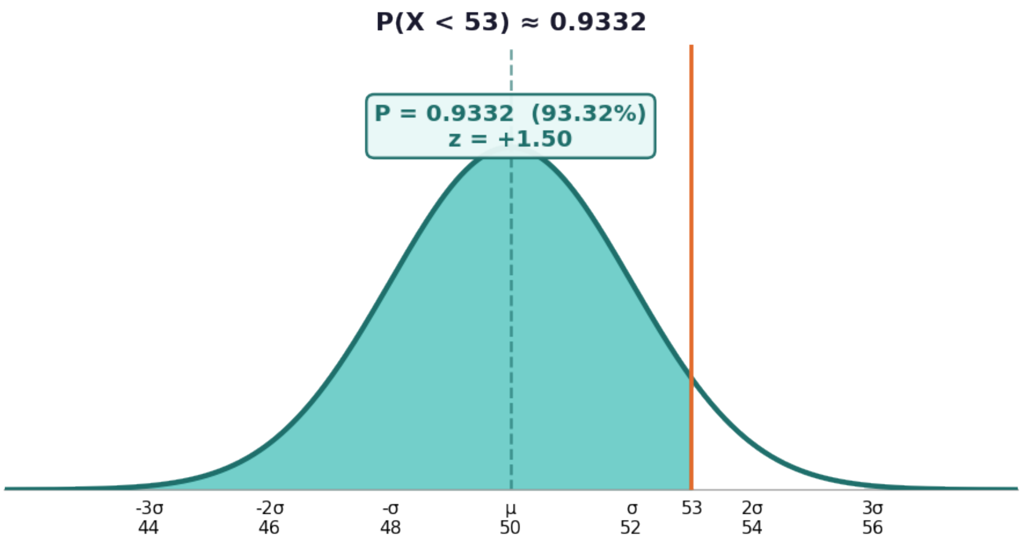
Answer Key

1. Answer: P ≈ 0.9332 (93.32%)



- Identify parameters: $\mu = 50$, $\sigma = 2$, $x = 53$
- Compute $z = (53 - 50) / 2 = 1.5$
- On TI-84: normalcdf(–1E99, 53, 50, 2)
- Result ≈ 0.9332, or about 93.32%

2. Answer: P ≈ 0.0912 (9.12%)

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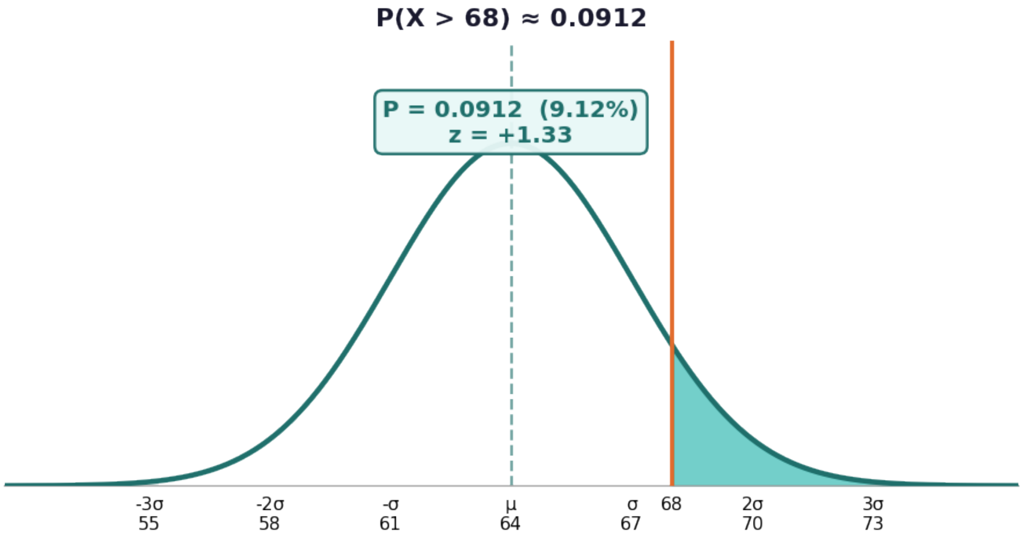

- Identify parameters: μ = 64, σ = 3, x = 68
- Compute z = (68 – 64) / 3 = 1.33
- On TI-84: normalcdf(68, 1E99, 64, 3)
- Result ≈ 0.0912, or about 9.12%

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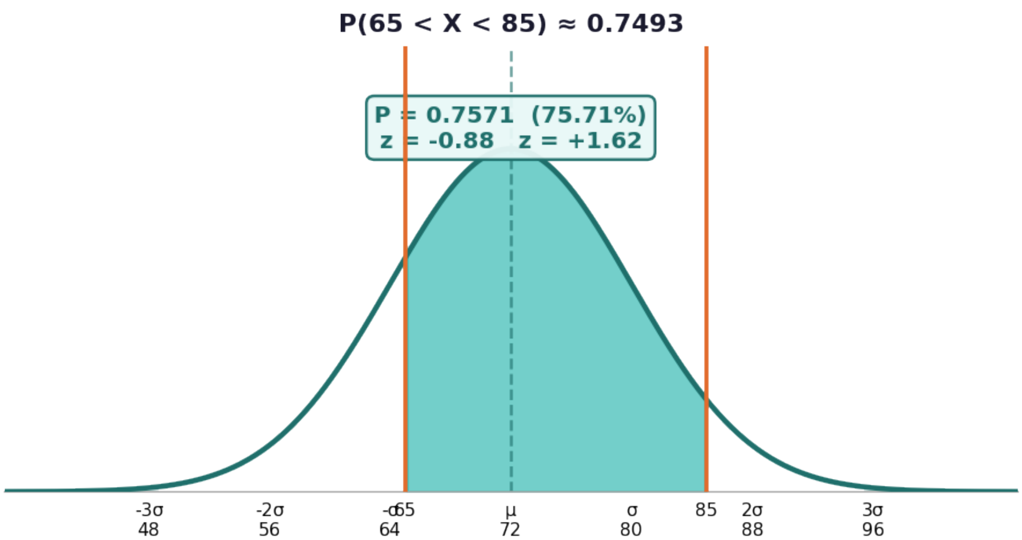
3. Answer: P ≈ 0.7493 (74.93%)



- Identify parameters: μ = 72, σ = 8
- Compute z■ = (65 – 72) / 8 = –0.875 and z■ = (85 – 72) / 8 = 1.625
- On TI-84: normalcdf(65, 85, 72, 8)
- Result ≈ 0.7493, or about 74.93%

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4. Answer: P ≈ 0.1588 (15.88%)

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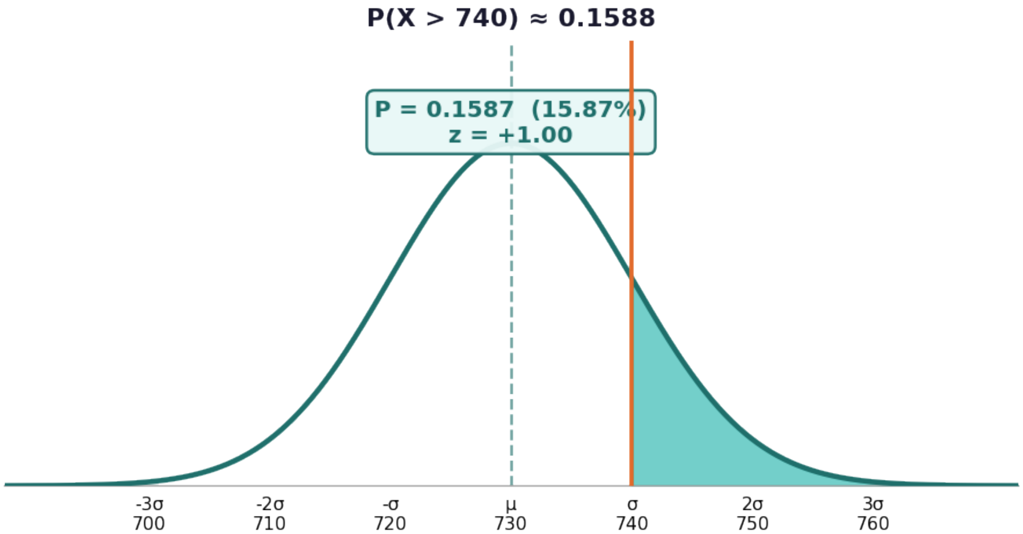

- Identify parameters: μ = 730, σ = 52, n = 27
- Compute standard error: σ_x■ = 52 / √27 ≈ 10.008
- On TI-84: normalcdf(740, 1E99, 730, 52/√(27))
- Result ≈ 0.1588, meaning there is approximately a 15.88% chance

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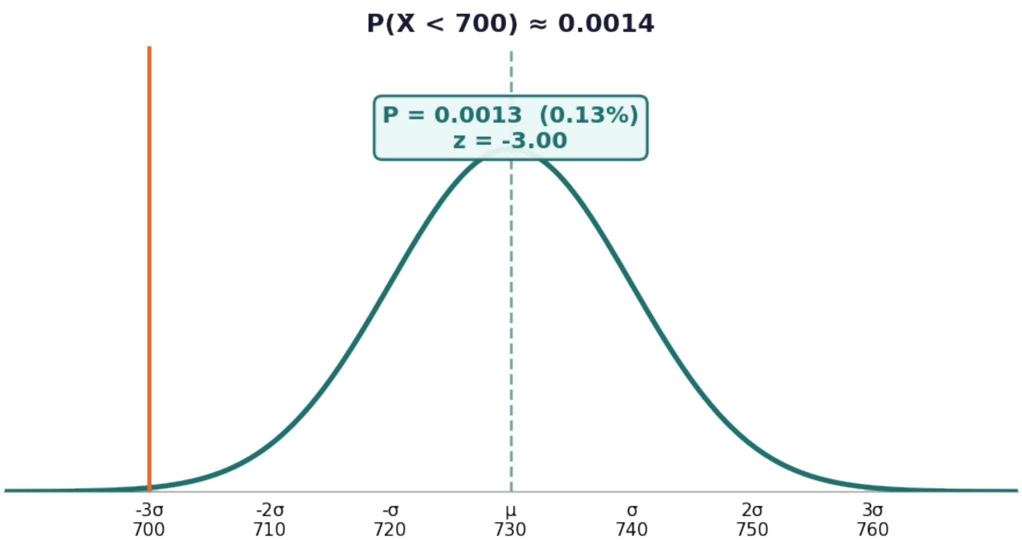
5. Answer: P ≈ 0.0014 (0.14%)



- Identify parameters: μ = 730, σ = 52, n = 27
- Compute standard error: σ_x■ = 52 / √27 ≈ 10.008
- On TI-84: normalcdf(–1E99, 700, 730, 52/√(27))
- Result ≈ 0.0014, meaning less than 1% probability

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6. Answer: P ≈ 0.6426 (64.26%)

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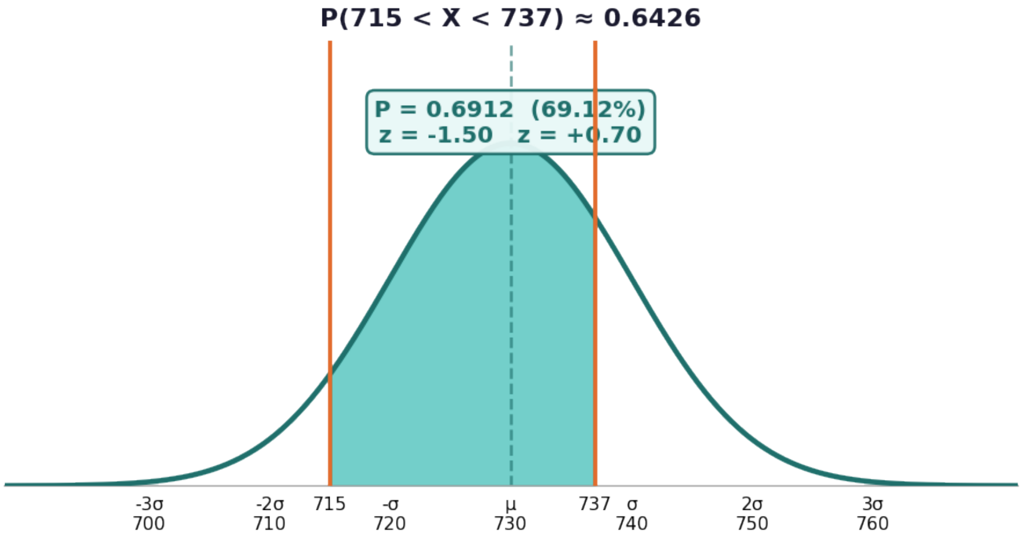

- Identify parameters: μ = 730, σ = 52, n = 27
- Compute standard error: σ_x■ = 52 / √27 ≈ 10.008
- z■ = (715 – 730) / 10.008 ≈ –1.499, z■ = (737 – 730) / 10.008 ≈ 0.699
- On TI-84: normalcdf(715, 737, 730, 52/√(27))
- Result ≈ 0.6426, or approximately 64.26%

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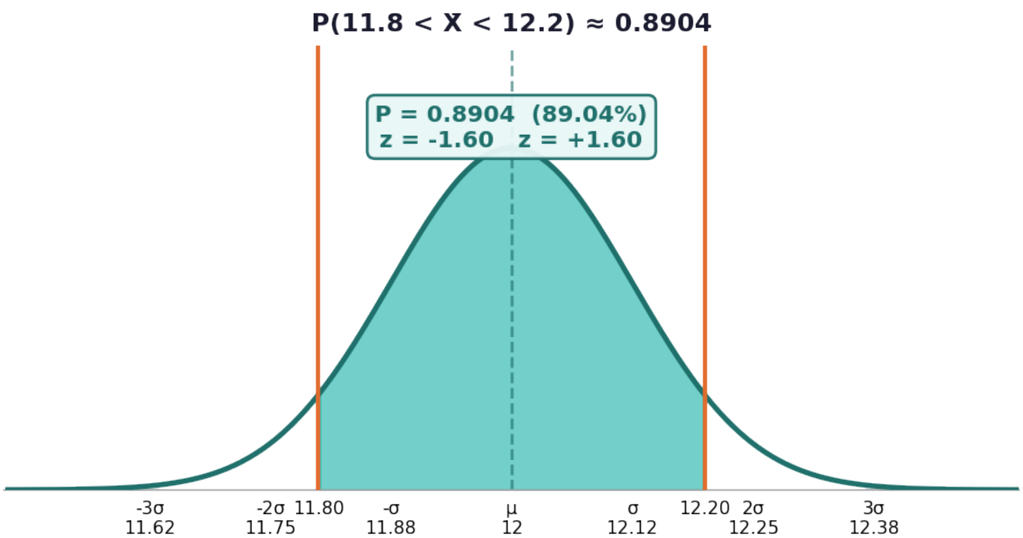
7. Answer: P ≈ 0.8904 (89.04%)



- Identify parameters: μ = 12, σ = 0.5, n = 16
- Compute standard error: σ_x■ = 0.5 / √16 = 0.5 / 4 = 0.125
- z■ = (11.8 – 12) / 0.125 = –1.6, z■ = (12.2 – 12) / 0.125 = 1.6
- On TI-84: normalcdf(11.8, 12.2, 12, 0.5/√(16))
- Result ≈ 0.8904, or approximately 89.04%

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8. Answer: P ≈ 0.0936 (9.36%)

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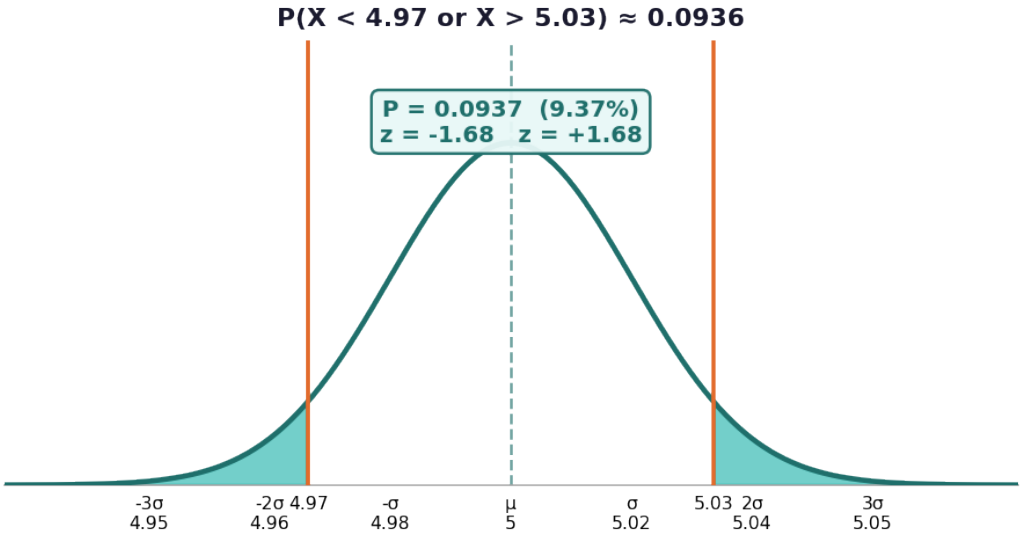

- Identify parameters: μ = 5, σ = 0.08, n = 20
- Compute standard error: σ_x■ = 0.08 / √20 ≈ 0.01789
- z■ = (4.97 – 5) / 0.01789 ≈ –1.677, z■ = (5.03 – 5) / 0.01789 ≈ 1.677
- On TI-84: P = normalcdf(–1E99, 4.97, 5, 0.08/√(20)) + normalcdf(5.03, 1E99, 5, 0.08/√(20))
- Alternatively: P = 1 – normalcdf(4.97, 5.03, 5, 0.08/√(20)) ≈ 0.0936

9. Answer: See completed table

Scenario	Sample Size (n)	Condition	Probability
A	27	P(X̄ > 740)	≈ 0.1588
B	27	P(X̄ < 700)	≈ 0.0014
C	50	P(X̄ > 740)	≈ 0.0862
D	50	P(X̄ < 700)	≈ 0.0001
E	100	P(X̄ > 740)	≈ 0.0274

- For each scenario, standard error = 52 / √n
- Scenario A (n=27): normalcdf(740, 1E99, 730, 52/√27) ≈ 0.1588
- Scenario B (n=27): normalcdf(–1E99, 700, 730, 52/√27) ≈ 0.0014
- Scenario C (n=50): normalcdf(740, 1E99, 730, 52/√50) ≈ 0.0862
- Scenario D (n=50): normalcdf(–1E99, 700, 730, 52/√50) ≈ 0.0001
- Scenario E (n=100): normalcdf(740, 1E99, 730, 52/√100) ≈ 0.0274
- Notice: as n increases, the sampling distribution becomes narrower and extreme tail probabilities decrease

10. Answer: (a) P ≈ 0.0308 (3.08%) (b) P ≈ 0.8706 (87.06%) (c) A larger n reduces the standard error, making it even less likely for the sample mean to exceed 75,000.

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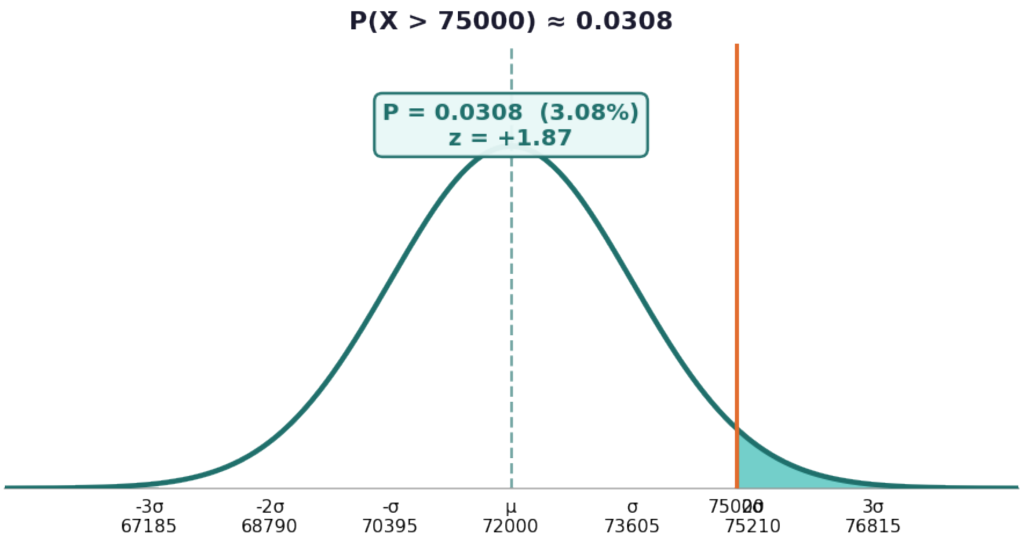

- Identify parameters: μ = 72000, σ = 9500, n = 35
- Compute standard error: σ_x■ = 9500 / √35 ≈ 1605.7
- Part (a): z = (75000 – 72000) / 1605.7 ≈ 1.868; normalcdf(75000, 1E99, 72000, 9500/√35) ≈ 0.0308
- Part (b): z■ = (69000 – 72000) / 1605.7 ≈ –1.868, z■ = (74500 – 72000) / 1605.7 ≈ 1.557; normalcdf(69000, 74500, 72000, 9500/√35) ≈ 0.8706
- Part (c): With n = 100, σ_x■ = 9500/√100 = 950, which is smaller than 1605.7, so the distribution is narrower and P(X■ > 75000) would decrease further (≈ 0.0008)

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