Normal Distribution & Sampling Distribution Probability

Statistics Worksheet · Grades 11–12 / Introductory College Statistics

Name: ________________________________ Date: ___________________

Learning Objectives

- Calculate z-scores using the sampling distribution formula z = (x■ – μ) / (σ / √n)
- Use the standard normal (Z) table to find probabilities for left-tail, right-tail, and between regions
- Interpret probabilities in context of real-world sampling problems

Problems

1. The GPAs of students at a university are normally distributed with a mean of 3.05 and a standard deviation of 0.29. A random sample of 20 students is selected. Find the z-score corresponding to a sample mean GPA of 2.98.

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

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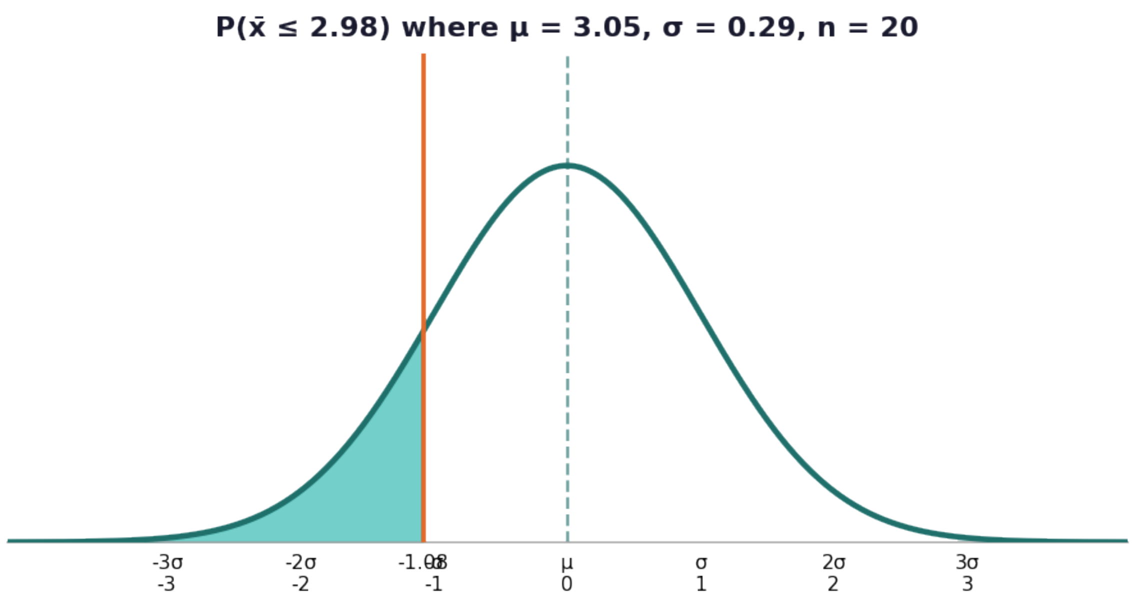
2. Using the same university GPA distribution (mean = 3.05, standard deviation = 0.29, sample size = 20), find the probability that the sample mean GPA is less than or equal to 2.98. Use the Z-table and shade the appropriate region.



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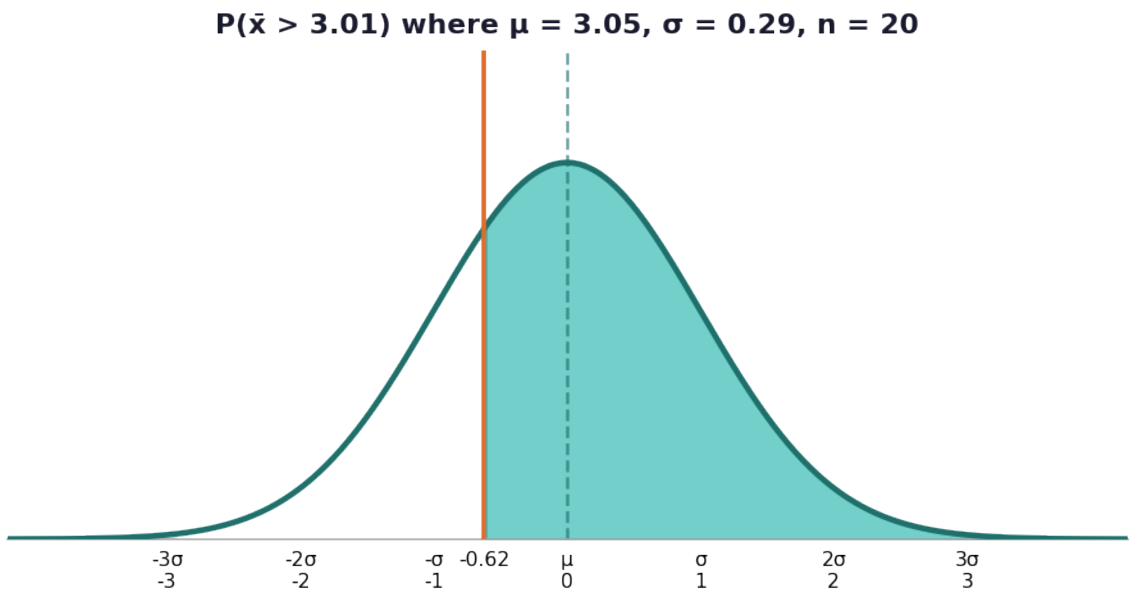
3. Using the same university GPA distribution (mean = 3.05, standard deviation = 0.29, sample size = 20), find the z-score corresponding to a sample mean GPA of 3.01.

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

4. Using the same university GPA distribution (mean = 3.05, standard deviation = 0.29, sample size = 20), find the probability that the sample mean GPA is greater than 3.01. Use the Z-table.



5. Using the same university GPA distribution (mean = 3.05, standard deviation = 0.29, sample size = 20), find the probability that the sample mean GPA is between 2.90 and 3.07. First compute both z-scores, then use the Z-table.

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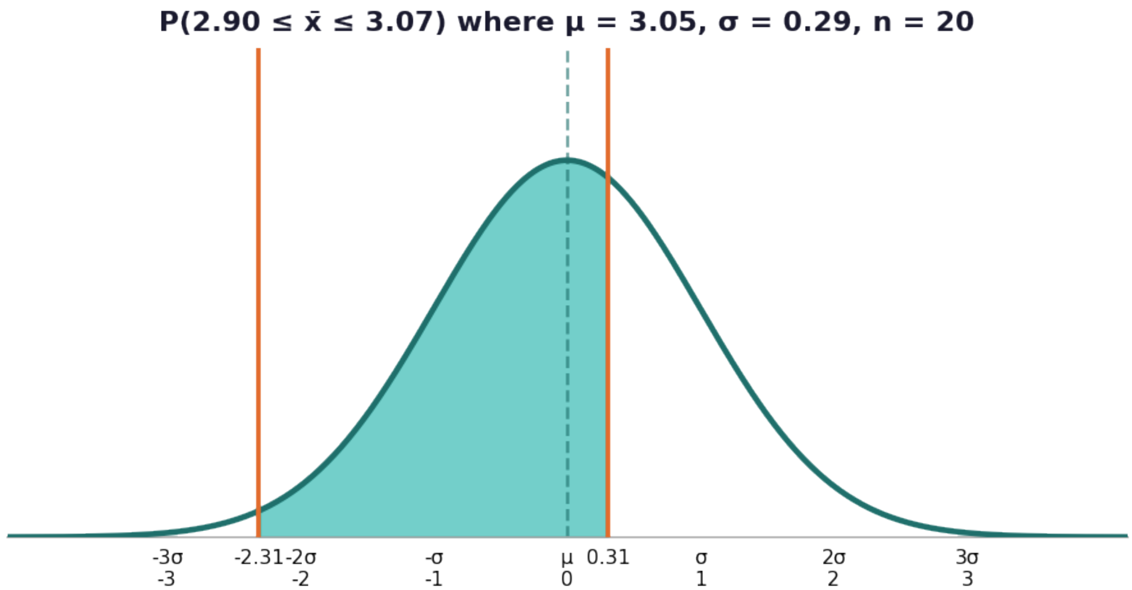
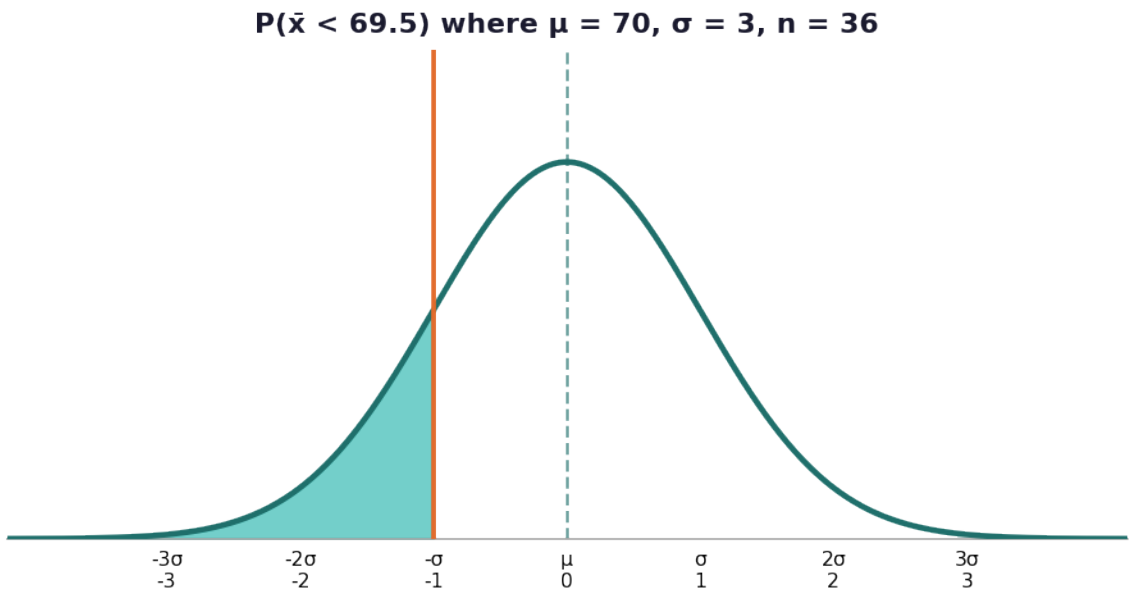


6. Heights of adult males are normally distributed with a mean of 70 inches and a standard deviation of 3 inches. A random sample of 36 men is taken. Find the probability that the sample mean height is less than 69.5 inches.



7. The daily caffeine intake of college students is normally distributed with a mean of 200 mg and a standard deviation of 40 mg. A sample of 25 students is selected. Find the probability that the sample mean caffeine intake is greater than 210 mg.

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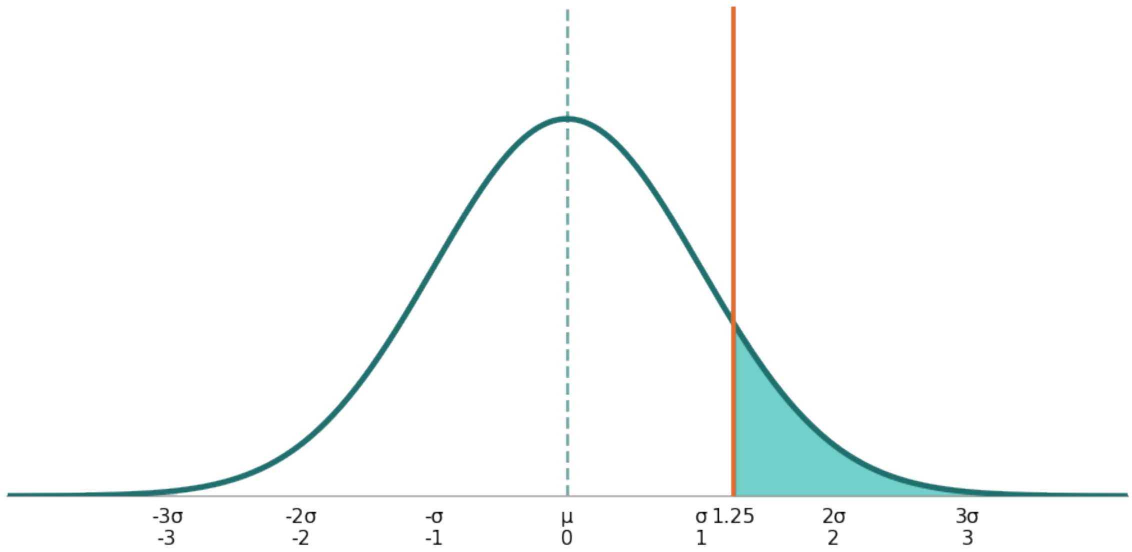
P(x̄ > 210) where μ = 200, σ = 40, n = 25



8. The table below shows z-scores and their corresponding left-tail probabilities from a Z-table. A student computed three z-scores for a sampling problem. Fill in the missing probabilities in the table, and identify which z-score gives the largest left-tail probability.

Z-Score	Left-Tail Probability (from Z-Table)	Greater or Less than 50%?
-1.45		
0.85		
2.10		

9. Scores on a standardized exam are normally distributed with a mean of 520 and a standard deviation of 80. A random sample of 64 students is drawn. Find the probability that the sample mean score falls between 505 and 535. Express your answer as a percentage rounded to two decimal places.

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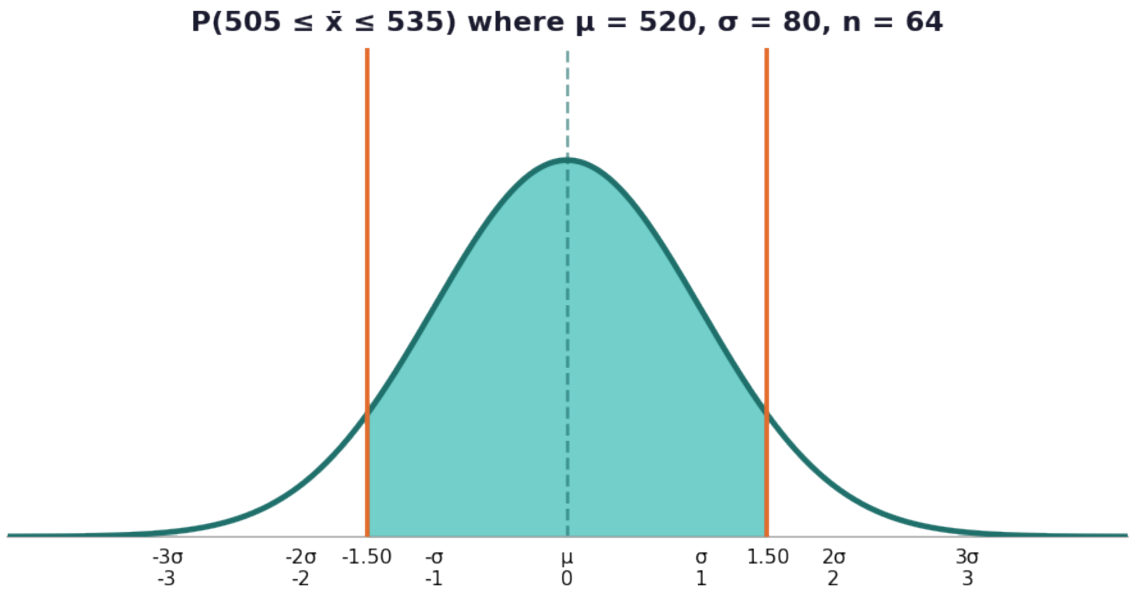
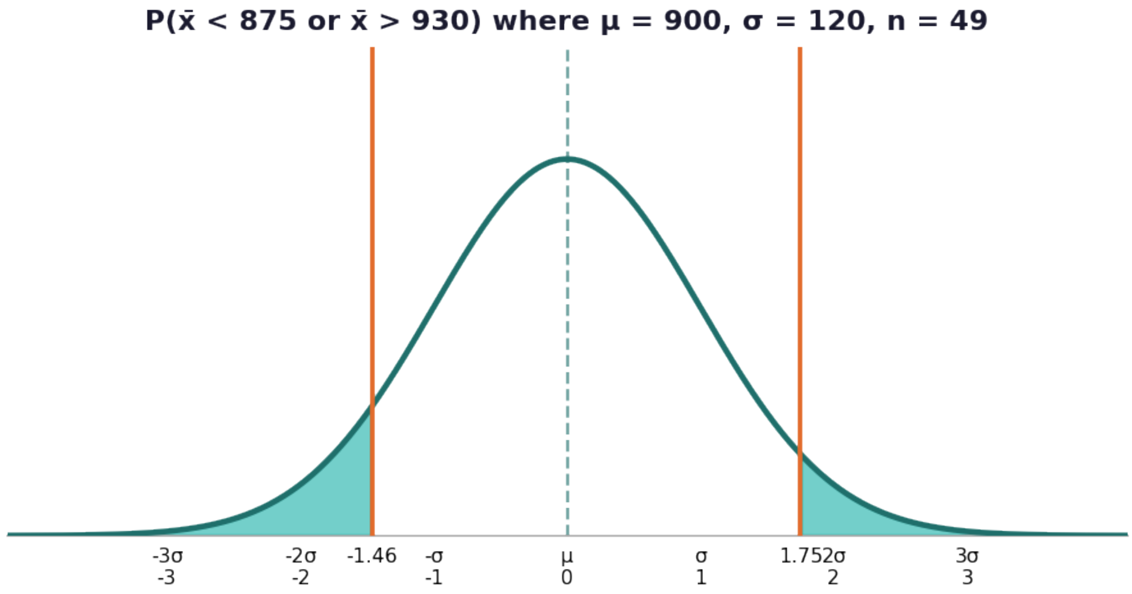


10. A nutritionist claims that the average sodium content per meal at a restaurant chain is 900 mg with a standard deviation of 120 mg. A health inspector randomly samples 49 meals. Find the probability that the sample mean sodium content is either less than 875 mg or greater than 930 mg. Round your final answer to four decimal places.



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Normal Distribution & Sampling Distribution Probability — Answer Key

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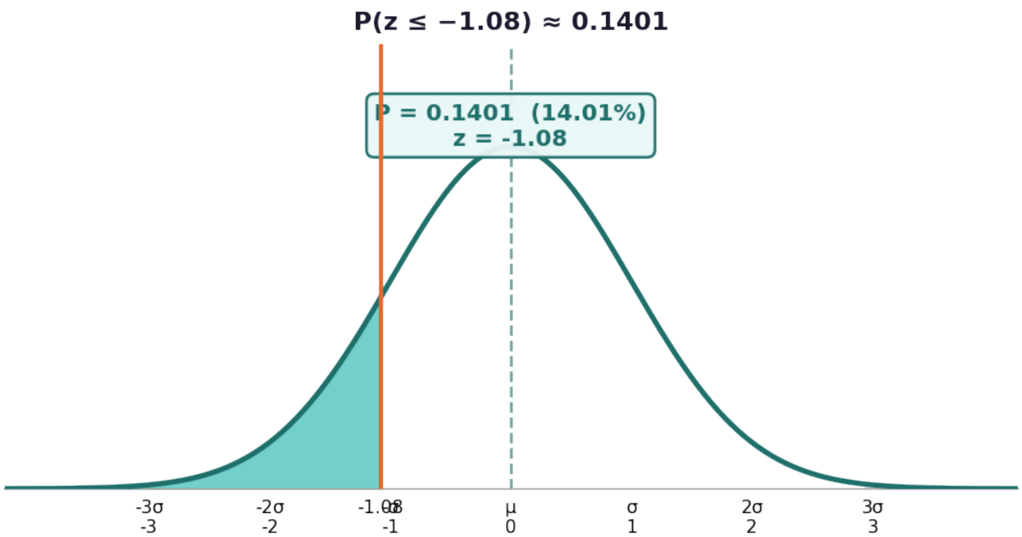
Answer Key

1. Answer: $z \approx -1.08$

- Identify values: $\bar{x} = 2.98$, $\mu = 3.05$, $\sigma = 0.29$, $n = 20$
- Compute $\sigma/\sqrt{n} = 0.29 / \sqrt{20} = 0.29 / 4.4721 \approx 0.0648$
- $z = (2.98 - 3.05) / 0.0648 = -0.07 / 0.0648 \approx -1.08$
- The z-score is approximately –1.08

2. Answer: $P \approx 0.1401$ (14.01%)



- From Problem 1, $z = -1.08$
- Look up $z = -1.08$ on the negative Z-table
- Find the row for –1.0 and column for 0.08
- The table value at $z = -1.08$ is 0.1401
- $P(\bar{x} \le 2.98) \approx 0.1401$, or about 14.01%

3. Answer: $z \approx -0.62$

- Identify values: $\bar{x} = 3.01$, $\mu = 3.05$, $\sigma = 0.29$, $n = 20$
- Compute $\sigma/\sqrt{n} = 0.29 / \sqrt{20} \approx 0.0648$
- $z = (3.01 - 3.05) / 0.0648 = -0.04 / 0.0648 \approx -0.62$
- The z-score is approximately –0.62

4. Answer: $P \approx 0.7324$ (73.24%)

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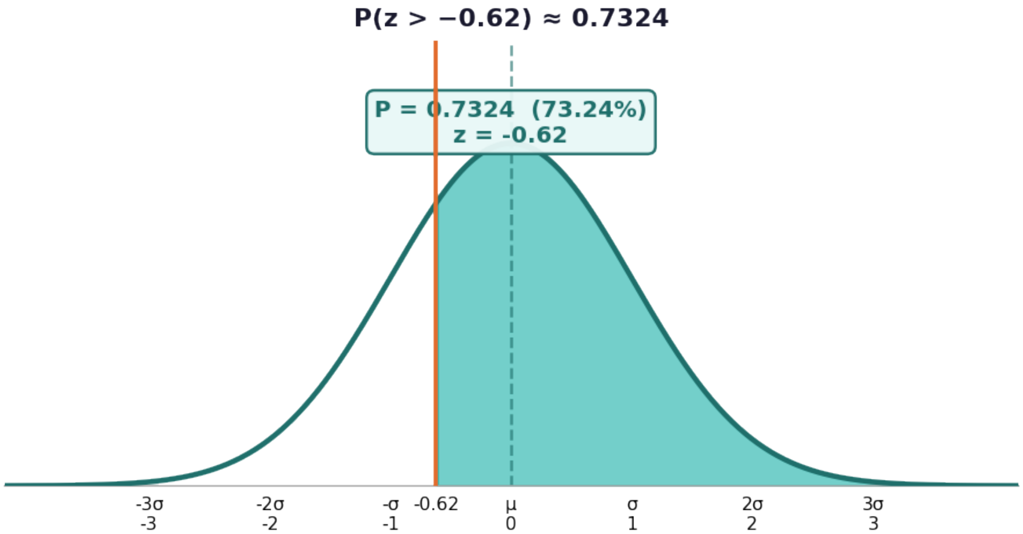

- From Problem 3, z = –0.62
- Look up z = –0.62 on the negative Z-table; table gives 0.2676
- Since we want P(z > –0.62), use the complement: 1 – 0.2676 = 0.7324
- P(x■ > 3.01) ≈ 0.7324, or about 73.24%

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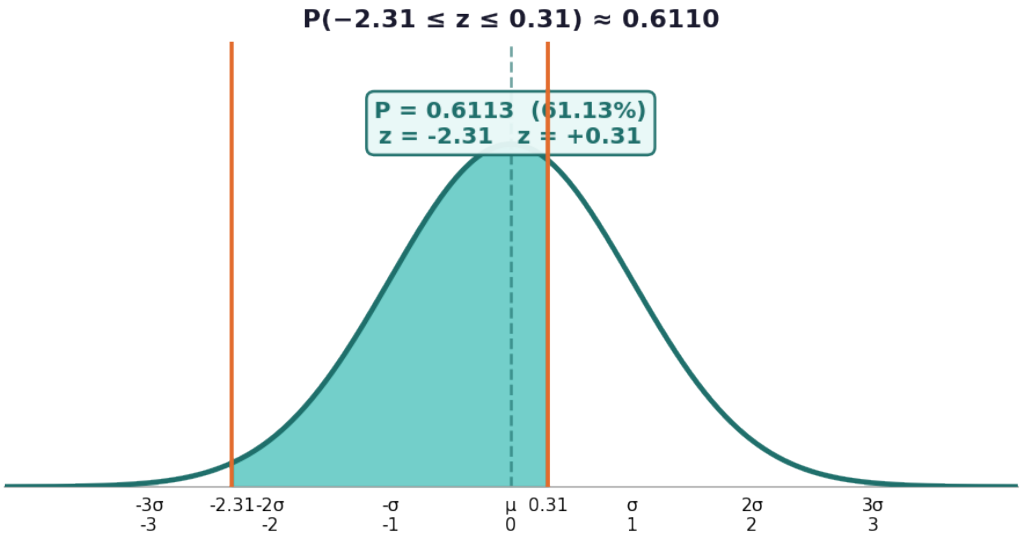
5. Answer: P ≈ 0.6110 (61.10%)



- σ/√n = 0.29 / √20 ≈ 0.0648
- z■ = (2.90 – 3.05) / 0.0648 = –0.15 / 0.0648 ≈ –2.31
- z■ = (3.07 – 3.05) / 0.0648 = 0.02 / 0.0648 ≈ 0.31
- P(z < 0.31) ≈ 0.6217 (positive Z-table)
- P(z < –2.31) ≈ 0.0104 (negative Z-table)
- P(–2.31 ≤ z ≤ 0.31) = 0.6217 – 0.0104 = 0.6113 ≈ 0.6110 (61.10%)

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6. Answer: P ≈ 0.1587 (15.87%)

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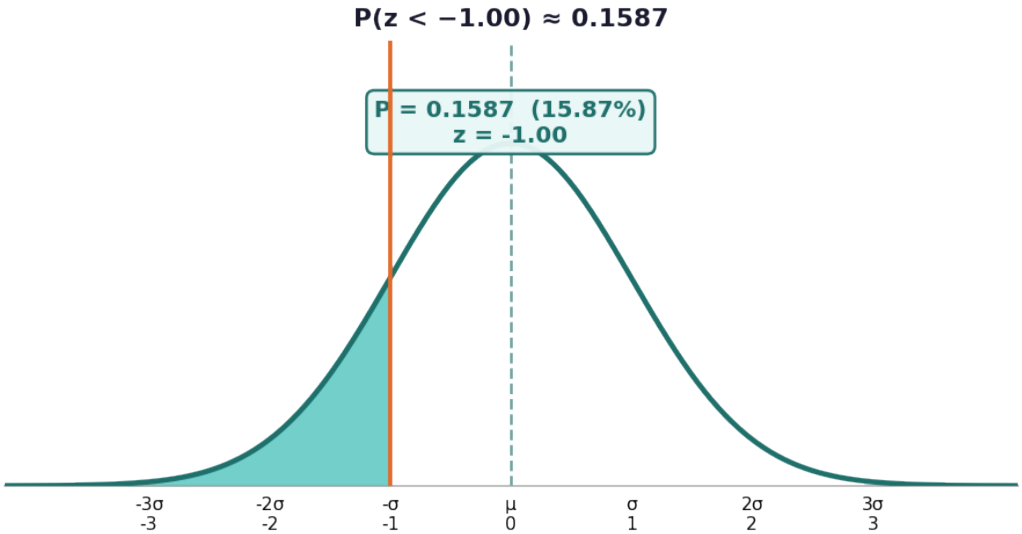

- Identify values: x■ = 69.5, μ = 70, σ = 3, n = 36
- σ/√n = 3 / √36 = 3 / 6 = 0.5
- z = (69.5 – 70) / 0.5 = –0.5 / 0.5 = –1.00
- Look up z = –1.00 in the negative Z-table: 0.1587
- P(x■ < 69.5) ≈ 0.1587 (15.87%)

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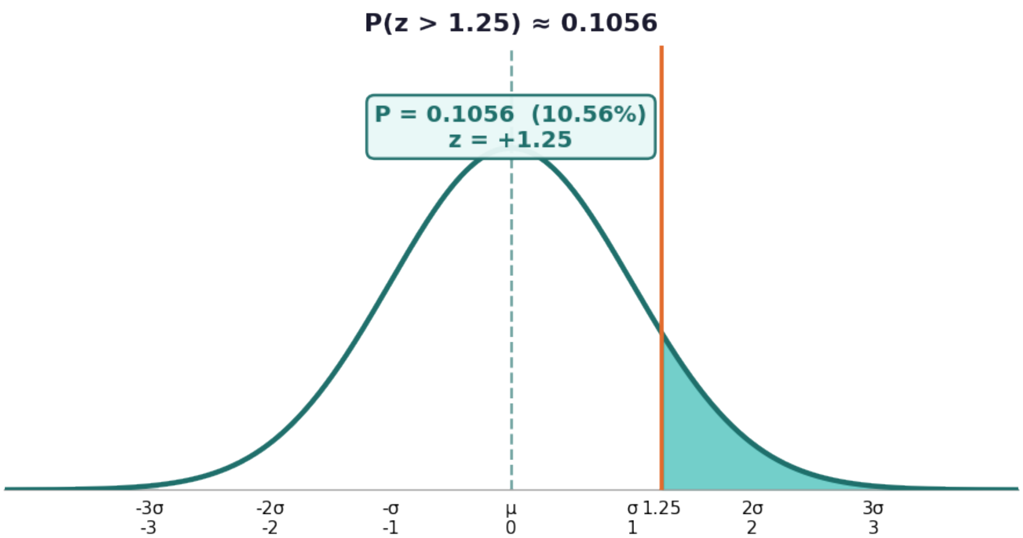
7. Answer: P ≈ 0.1056 (10.56%)



- Identify values: x■ = 210, μ = 200, σ = 40, n = 25
- σ/√n = 40 / √25 = 40 / 5 = 8
- z = (210 – 200) / 8 = 10 / 8 = 1.25
- Look up z = 1.25 in the positive Z-table: 0.8944
- P(z > 1.25) = 1 – 0.8944 = 0.1056 (10.56%)

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8. Answer: z = –1.45 → 0.0735; z = 0.85 → 0.8023; z = 2.10 → 0.9821. Largest: z = 2.10

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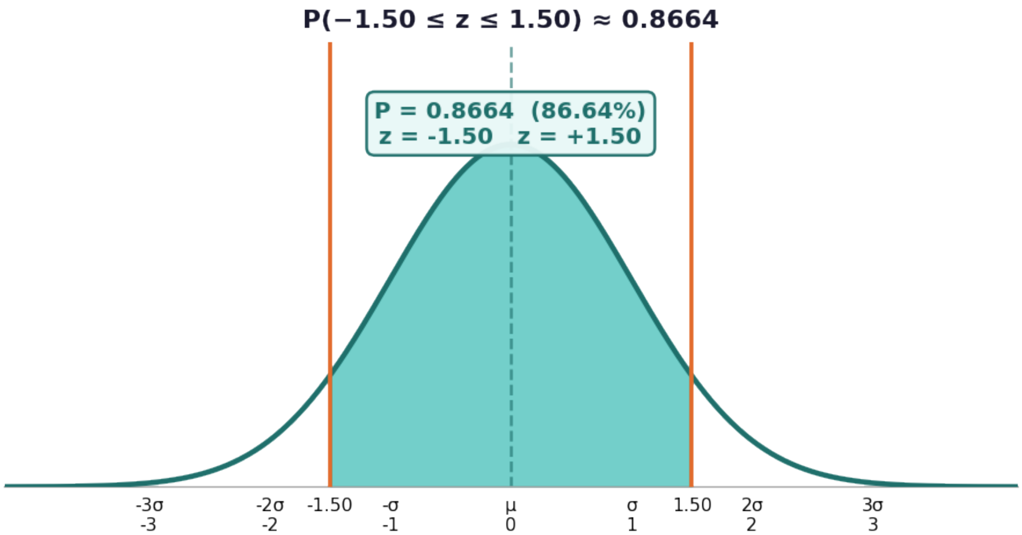
Z-Score	Left-Tail Probability (from Z-Table)	Greater or Less than 50%?
-1.45	0.0735	Less than 50%
0.85	0.8023	Greater than 50%
2.10	0.9821	Greater than 50%

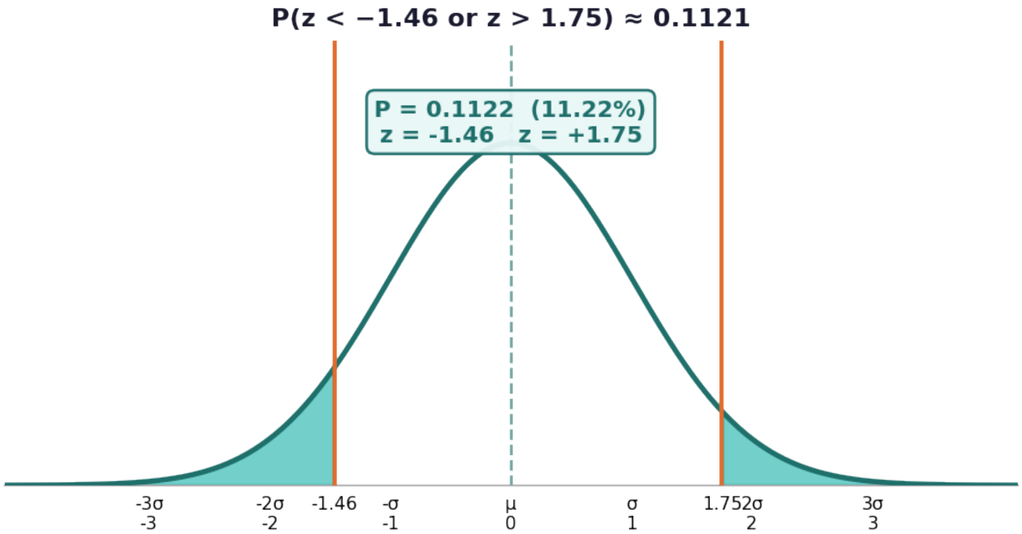
- For z = –1.45: look up row –1.4, column 0.05 on negative Z-table → 0.0735
- For z = 0.85: look up row 0.8, column 0.05 on positive Z-table → 0.8023
- For z = 2.10: look up row 2.1, column 0.00 on positive Z-table → 0.9821
- Negative z-scores give probabilities below 50%; positive z-scores above 50%
- The largest left-tail probability is at z = 2.10 (0.9821)

9. Answer: P ≈ 0.8664 (86.64%)



- σ/√n = 80 / √64 = 80 / 8 = 10
- z■ = (505 – 520) / 10 = –15 / 10 = –1.50
- z■ = (535 – 520) / 10 = 15 / 10 = 1.50
- P(z < 1.50) = 0.9332 (positive Z-table)
- P(z < –1.50) = 0.0668 (negative Z-table)
- P(–1.50 ≤ z ≤ 1.50) = 0.9332 – 0.0668 = 0.8664 (86.64%)

10. Answer: P ≈ 0.1121 (11.21%)



- $\sigma/\sqrt{n}$ = 120 / √49 = 120 / 7 ≈ 17.14
- z■ = (875 – 900) / 17.14 = –25 / 17.14 ≈ –1.46
- z■ = (930 – 900) / 17.14 = 30 / 17.14 ≈ 1.75
- P(z < –1.46) ≈ 0.0721 (negative Z-table)
- P(z > 1.75) = 1 – P(z < 1.75) = 1 – 0.9599 = 0.0401
- P(outside) = 0.0721 + 0.0401 = 0.1122 ≈ 0.1121 (11.21%)