Confidence Intervals: Estimating the Population Mean

Statistics Worksheet · Grade 11–12

Name: ________________________________ Date: ___________________

Learning Objectives

- Understand what a confidence interval is and how it estimates the true population mean
- Calculate confidence intervals using sample statistics and z-scores
- Interpret confidence levels and explain what they mean in context

Problems

1. A researcher wants to estimate the average weight of rabbits in a large population. She knows the population mean is unknowable. Which of the following best describes a confidence interval?

2. A sample of 30 rabbits has a mean weight of 3.2 lb and a population standard deviation of 0.8 lb. Identify the point estimate of the population mean.

$$\bar{x} = 3.2 \text{ lb}, \quad \sigma = 0.8 \text{ lb}, \quad n = 30$$

3. Three samples were taken from a rabbit population. Sample 1 (n = 30) gave x-bar = 3.2 lb, Sample 2 (n = 245) gave x-bar = 4.2 lb, and Sample 3 (n = 326) gave x-bar = 5.9 lb. Which sample is most likely to produce the most reliable estimate of the population mean, and why?

4. Calculate the standard error of the mean for a sample of 50 rabbits if the population standard deviation is 1.4 lb.

$$SE = \frac{\sigma}{\sqrt{n}}, \quad \sigma = 1.4, \quad n = 50$$

5. A confidence interval for rabbit weights is reported as (3.5, 5.5) lb at an 80% confidence level. What is the margin of error for this interval?

$$CI = (3.5,\ 5.5)$$

Scan to watch

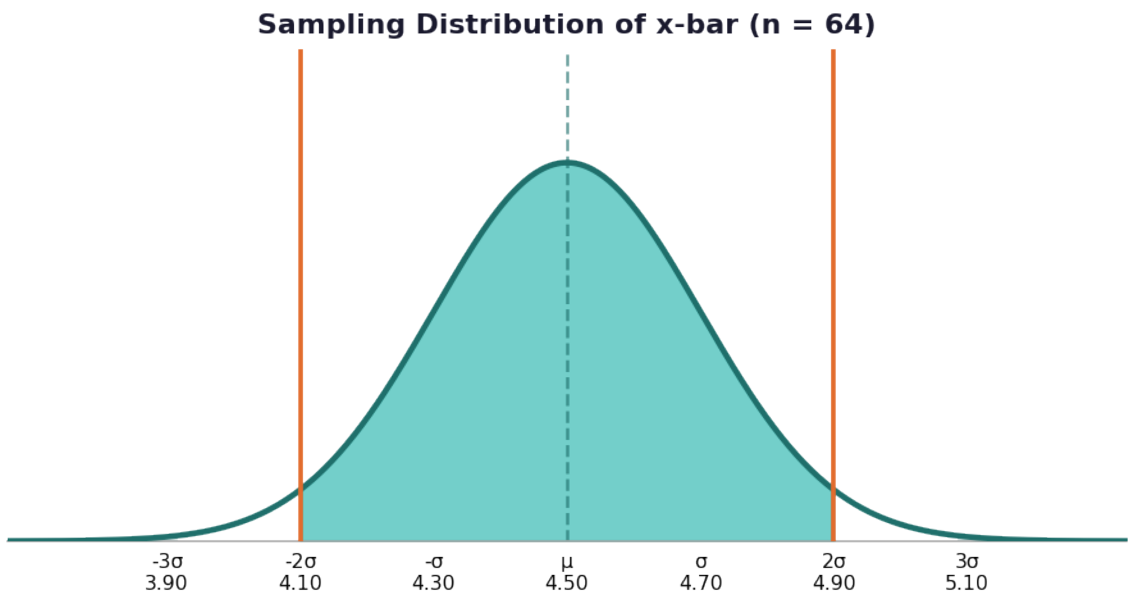
6. The table below shows results from repeated samples of size 50 from a rabbit population. The confidence interval was found to be (3.5, 5.5) lb. Determine which sample means fall INSIDE the confidence interval and which fall OUTSIDE.

Sample	Sample Mean (lb)	Inside CI (3.5, 5.5)?
1	4.1	
2	5.2	
3	3.1	
4	4.8	
5	5.7	

7. A sample of 64 rabbits has a mean weight of 4.5 lb. The population standard deviation is known to be 1.6 lb. Construct a 95% confidence interval for the true population mean. Use z-star = 1.96.

$$CI = \bar{x} \pm z^* \cdot \frac{\sigma}{\sqrt{n}}$$

8. The shaded region below represents the area under a normal distribution for a sample mean. The population of rabbit weights has a mean of 4.5 lb and a standard deviation of 1.6 lb with n = 64. Find the probability that a randomly selected sample mean falls between 4.1 and 4.9 lb.

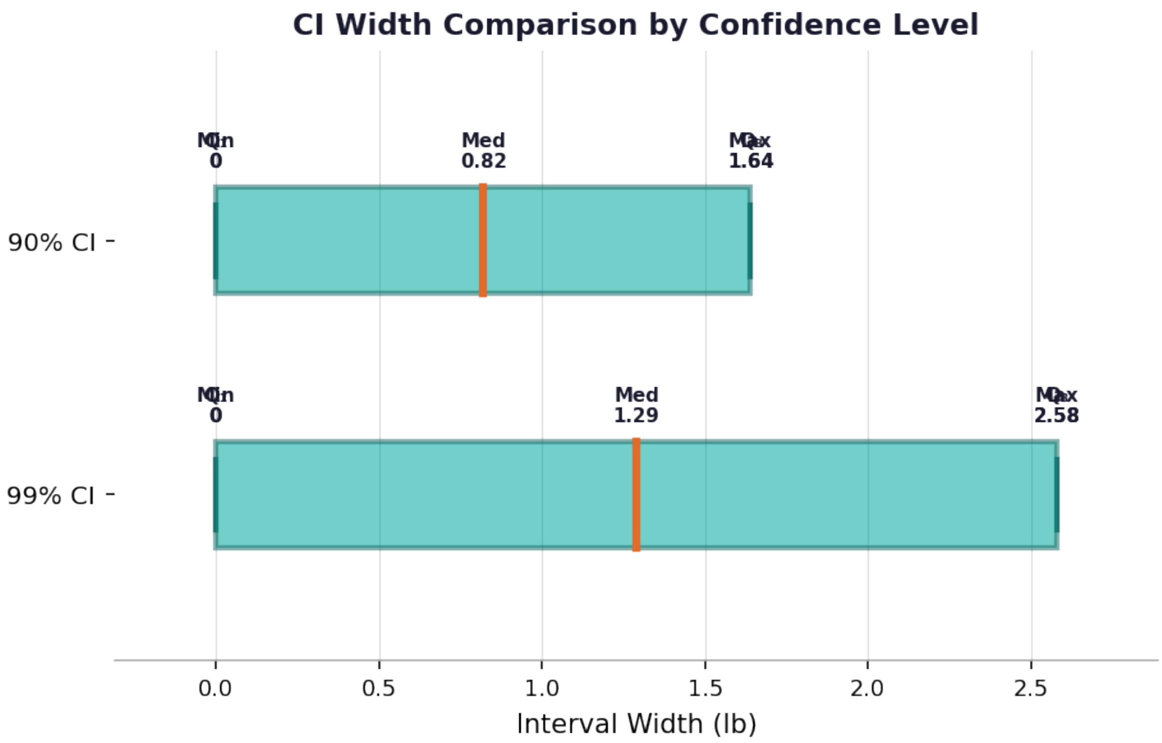


Scan to watch

9. A researcher wants to estimate the mean weight of rabbits within a margin of error of 0.25 lb at a 99% confidence level. The population standard deviation is 1.5 lb. Use z-star = 2.576. What minimum sample size is required?

$$n = \left(\frac{z^* \cdot \sigma}{E}\right)^2$$

10. Two researchers study the same rabbit population. Researcher A uses a 90% confidence level and Researcher B uses a 99% confidence level, both with the same sample size and standard deviation. The box plot below shows the widths of their confidence intervals. Researcher A reports CI width = 1.64 lb and Researcher B reports CI width = 2.58 lb. Explain which interval is wider, why this occurs, and what trade-off exists between confidence level and interval precision.



Scan to watch

Confidence Intervals: Estimating the Population Mean — Answer Key

Statistics Worksheet · Grade 11–12

Answer Key

1. Answer: A range of values computed from a sample that is likely to contain the true population mean

- A confidence interval is NOT a single value — it is a range (lower bound, upper bound).
- It is calculated from sample data and is designed to capture the true population mean with a stated level of confidence.
- Answer: A range of values computed from a sample that is likely to contain the true population mean.

2. Answer: Point estimate = 3.2 lb

- The point estimate of the population mean (μ) is simply the sample mean ($\bar{x}$).
- $\bar{x}$ = 3.2 lb
- Therefore, the point estimate is 3.2 lb.

3. Answer: Sample 3 (n = 326); larger sample size reduces sampling error and produces a more reliable estimate

- As sample size increases, the standard error of the mean decreases: $SE = \sigma / \sqrt{n}$.
- Larger n → smaller SE → the sample mean is closer to the true population mean.
- Sample 3 has the largest n (326), so it provides the most reliable estimate.

4. Answer: SE ≈ 0.198 lb

- $SE = \sigma / \sqrt{n} = 1.4 / \sqrt{50}$
- $\sqrt{50} \approx 7.071$
- $SE = 1.4 / 7.071 \approx 0.198$ lb

5. Answer: Margin of error = 1.0 lb

- The margin of error (E) is half the width of the confidence interval.
- Width = Upper bound – Lower bound = 5.5 – 3.5 = 2.0
- E = Width / 2 = 2.0 / 2 = 1.0 lb

6. Answer: Inside: Samples 1, 2, 4; Outside: Samples 3, 5

Sample	Sample Mean (lb)	Inside CI (3.5, 5.5)?
1	4.1	Yes
2	5.2	Yes
3	3.1	No

Scan to watch

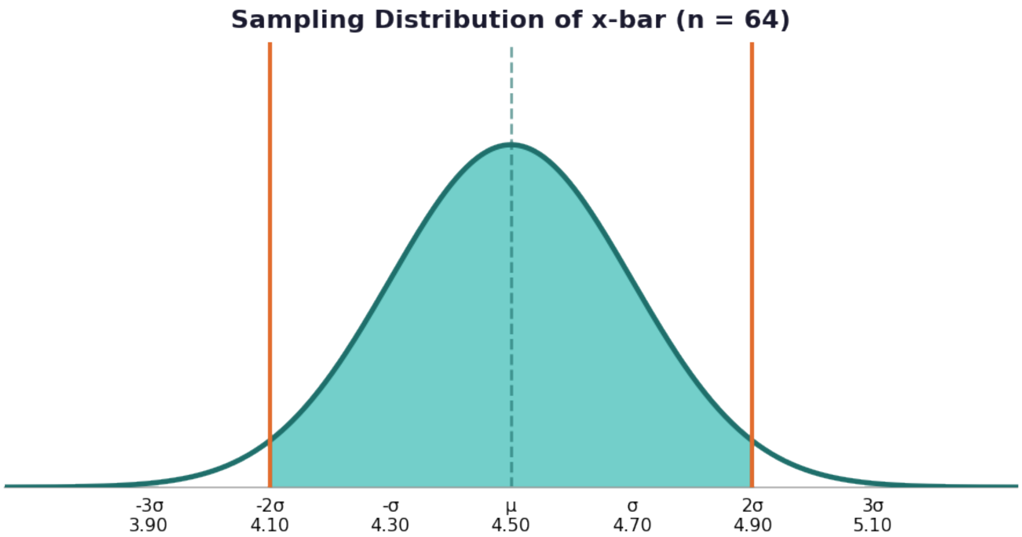
Sample	Sample Mean (lb)	Inside CI (3.5, 5.5)?
4	4.8	Yes
5	5.7	No

- Check each mean: is $3.5 \le \bar{x} \le 5.5$?
- 4.1: Yes (inside) | 5.2: Yes (inside) | 3.1: No (below 3.5) | 4.8: Yes (inside) | 5.7: No (above 5.5)
- 3 out of 5 samples fall inside the interval.

7. Answer: 95% CI: (4.108, 4.892) lb

- $SE = \sigma / \sqrt{n} = 1.6 / \sqrt{64} = 1.6 / 8 = 0.2$
- Margin of error $E = z^* \times SE = 1.96 \times 0.2 = 0.392$
- Lower bound = 4.5 – 0.392 = 4.108
- Upper bound = 4.5 + 0.392 = 4.892
- 95% CI: (4.108, 4.892) lb

8. Answer: P ≈ 0.9545 (95.45%)



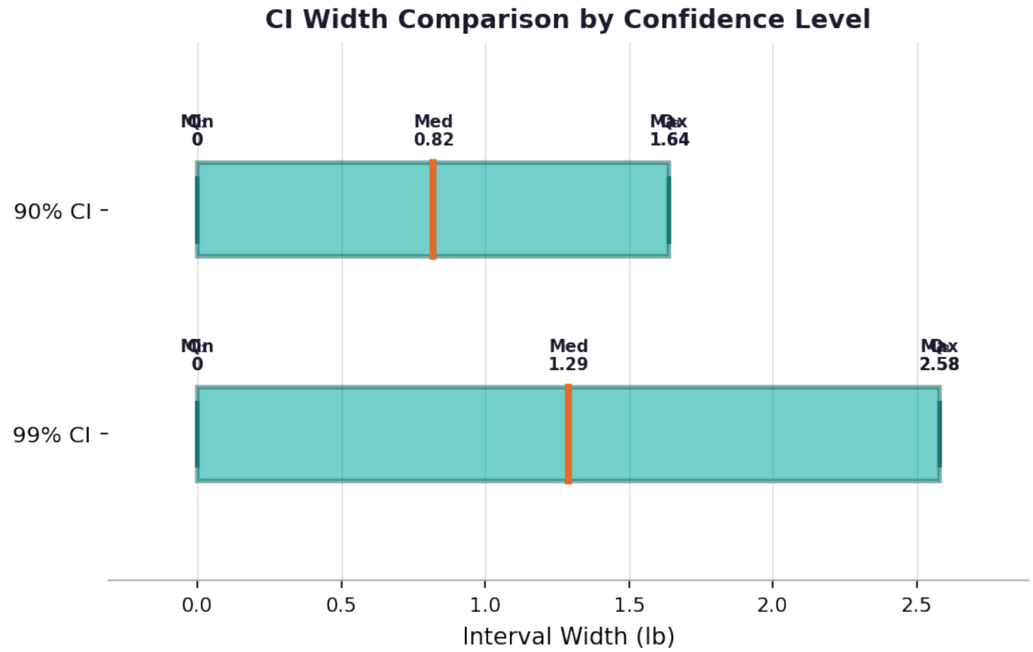
- $SE = \sigma / \sqrt{n} = 1.6 / 8 = 0.2$
- $z1 = (4.1 - 4.5) / 0.2 = -2.00$
- $z2 = (4.9 - 4.5) / 0.2 = 2.00$
- $P(-2 < z < 2) \approx 0.9545$
- There is approximately a 95.45% chance the sample mean falls between 4.1 and 4.9 lb.

9. Answer: Minimum n = 239

- Use the sample size formula: $n = (z^* \times \sigma / E)^2$
- $n = (2.576 \times 1.5 / 0.25)^2$
- $= (3.864 / 0.25)^2$
- $= (15.456)^2$
- = 238.887
- Always round UP to the next whole number: n = 239

Scan to watch

10. Answer: Researcher B's 99% CI is wider. Higher confidence requires a larger z*, which increases the margin of error, producing a wider but less precise interval.



- Margin of error $E = z^* \times (\sigma / \sqrt{n})$. For 90%, $z^* = 1.645$; for 99%, $z^* = 2.576$.
- Since z^* is larger for 99%, the margin of error is larger, making the CI wider.
- Researcher B's interval (2.58 lb wide) is wider than Researcher A's (1.64 lb wide).
- Trade-off: A higher confidence level gives more certainty that the interval captures μ, but the interval is less precise (wider range).
- A lower confidence level gives a narrower, more precise interval, but with less certainty of capturing μ.

Scan to watch