Confidence Intervals for Population Mean

Statistics Worksheet · Grade 11–12 / Introductory College Statistics

Name: ________________________________ Date: ___________________

Learning Objectives

- Identify and organize given values (sample size, sample mean, population standard deviation, confidence level) from word problems
- Verify conditions for constructing a valid confidence interval for a population mean
- Calculate the critical z-value, margin of error, and confidence interval using the formula x■ ± z* · (σ / √n)

Problems

1. A simple random sample of 100 students was taken from a normally distributed population. The sample mean score on a reading test was 520 points, and the population standard deviation is 80 points. Identify the values of n, x-bar, sigma, and the confidence level if you are constructing a 95% confidence interval.

$$n = ?, \quad \bar{x} = ?, \quad \sigma = ?, \quad C = ?$$

2. To construct a confidence interval for a population mean, two conditions must be verified. State both conditions and check whether they are satisfied for this scenario: An SRS of 200 adults was drawn from a normally distributed population.

3. Find the critical z-value (z-star) for a 90% confidence level. Use the formula (1 minus C) divided by 2 to find the tail area, then use the standard normal table or calculator.

$$z^* \text{ for } C = 0.90$$

4. Find the critical z-value (z-star) for a 99% confidence level.

$$z^* \text{ for } C = 0.99$$

5. A normally distributed population has a standard deviation of 50. An SRS of 400 people is taken with a sample mean of 300. Calculate the margin of error for a 95% confidence interval. Use z-star = 1.96.

$$E = z^* \cdot \frac{\sigma}{\sqrt{n}}$$

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6. Using the information from the video: an SRS of 500 students from a normally distributed population has a sample mean SAT math score of 461, and the population standard deviation is 100. Construct the 95% confidence interval for the population mean. Use z-star = 1.96.

$$\bar{x} \pm z^* \cdot \frac{\sigma}{\sqrt{n}}$$

7. A researcher takes an SRS of 64 employees from a normally distributed population. The sample mean salary is $45,000 and the population standard deviation is $8,000. Construct a 90% confidence interval for the true population mean salary. Use z-star = 1.645.

$$\bar{x} \pm z^* \cdot \frac{\sigma}{\sqrt{n}}$$

8. A 95% confidence interval for a population mean is reported as (112.4, 127.6). Find the sample mean and the margin of error from this interval.

$$\bar{x} = \frac{\text{Lower} + \text{Upper}}{2}, \quad E = \frac{\text{Upper} - \text{Lower}}{2}$$

9. A health researcher wants to estimate the mean cholesterol level of adults in a city. From a normally distributed population with a known standard deviation of 35 mg/dL, an SRS of 225 adults yields a sample mean of 198 mg/dL. Construct a 99% confidence interval and write a concluding statement. Use z-star = 2.576.

$$\bar{x} \pm z^* \cdot \frac{\sigma}{\sqrt{n}}$$

10. Compare the widths of the 90%, 95%, and 99% confidence intervals for a population with sigma = 120, constructed from an SRS of 900. The sample mean is 850. Calculate all three intervals and explain how the confidence level affects the width of the interval.

$$\bar{x} \pm z^* \cdot \frac{\sigma}{\sqrt{n}}, \quad z^*_{90} = 1.645, \ z^*_{95} = 1.96, \ z^*_{99} = 2.576$$

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Confidence Intervals for Population Mean — Answer Key

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Answer Key

1. Answer: n = 100, x-bar = 520, sigma = 80, C = 0.95

- Read the problem: sample size = 100, so n = 100.
- The sample mean is stated as 520, so x-bar = 520.
- The population standard deviation is given as 80, so sigma = 80.
- The confidence level is 95%, written as C = 0.95.

2. Answer: Condition 1: Sample is randomly selected — satisfied (SRS). Condition 2: Population is normally distributed — satisfied (stated in problem).

- Condition 1: The sample must be randomly selected. The problem states it is an SRS, so this condition is satisfied.
- Condition 2: The population must be normally distributed (or the sample size large enough for CLT). The problem states the population is normally distributed, so this condition is satisfied.
- Both conditions are met; it is valid to proceed with the confidence interval.

3. Answer: z* = 1.645

- Tail area = (1 – 0.90) / 2 = 0.10 / 2 = 0.05.
- We need the z-value where the cumulative area to the left is 1 – 0.05 = 0.95.
- From the standard normal table or calculator (inverse normal): z* = 1.645.

4. Answer: z* = 2.576

- Tail area = (1 – 0.99) / 2 = 0.01 / 2 = 0.005.
- We need the z-value where the cumulative area is 1 – 0.005 = 0.995.
- From the standard normal table or inverse normal on a calculator: z* = 2.576.

5. Answer: E = 4.9

- Identify values: z* = 1.96, sigma = 50, n = 400.
- Standard error = sigma / sqrt(n) = 50 / sqrt(400) = 50 / 20 = 2.5.
- Margin of error E = 1.96 × 2.5 = 4.9.

6. Answer: (452.23, 469.77)

- Given: x-bar = 461, z* = 1.96, sigma = 100, n = 500.
- Standard error = 100 / sqrt(500) = 100 / 22.36 ≈ 4.47.
- Margin of error E = 1.96 × 4.47 ≈ 8.77.
- Lower bound = 461 – 8.77 = 452.23.
- Upper bound = 461 + 8.77 = 469.77.
- The 95% confidence interval is (452.23, 469.77).

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7. Answer: ($43,355, $46,645)

- Given: x-bar = 45000, z* = 1.645, sigma = 8000, n = 64.
- Standard error = 8000 / sqrt(64) = 8000 / 8 = 1000.
- Margin of error E = 1.645 × 1000 = 1645.
- Lower bound = 45000 – 1645 = 43355.
- Upper bound = 45000 + 1645 = 46645.
- The 90% confidence interval is ($43,355, $46,645).

8. Answer: x-bar = 120, E = 7.6

- Sample mean = (112.4 + 127.6) / 2 = 240 / 2 = 120.
- Margin of error E = (127.6 – 112.4) / 2 = 15.2 / 2 = 7.6.
- Interpretation: The confidence interval is centered at 120 with a margin of error of 7.6.

9. Answer: (192.0, 204.0); We are 99% confident the true mean cholesterol level lies between 192.0 and 204.0 mg/dL.

- Given: x-bar = 198, z* = 2.576, sigma = 35, n = 225.
- Verify conditions: SRS (yes) and normally distributed population (yes).
- Standard error = 35 / sqrt(225) = 35 / 15 ≈ 2.333.
- Margin of error E = 2.576 × 2.333 ≈ 6.01.
- Lower bound = 198 – 6.01 = 191.99 ≈ 192.0.
- Upper bound = 198 + 6.01 = 204.01 ≈ 204.0.
- Conclusion: We are 99% confident the true mean cholesterol level lies between 192.0 and 204.0 mg/dL.

10. Answer: 90%: (843.42, 856.58); 95%: (842.16, 857.84); 99%: (839.71, 860.29). Higher confidence level → wider interval.

- Given: x-bar = 850, sigma = 120, n = 900. Standard error = 120 / sqrt(900) = 120 / 30 = 4.
- 90% CI: E = 1.645 × 4 = 6.58. Interval: (850 – 6.58, 850 + 6.58) = (843.42, 856.58). Width = 13.16.
- 95% CI: E = 1.96 × 4 = 7.84. Interval: (850 – 7.84, 850 + 7.84) = (842.16, 857.84). Width = 15.68.
- 99% CI: E = 2.576 × 4 = 10.30. Interval: (850 – 10.30, 850 + 10.30) = (839.70, 860.30). Width = 20.60.
- Conclusion: As the confidence level increases, the critical z-value increases, resulting in a larger margin of error and a wider confidence interval. Higher confidence comes at the cost of less precision.

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