Confidence Intervals for Population Mean Using the T-Distribution

Statistics Worksheet · Grade 11–12 / Introductory College Statistics

Name: ________________________________ Date: ___________________

Learning Objectives

- Identify when to use the t-distribution instead of the z-distribution for confidence intervals
- Calculate the sample mean, sample standard deviation, degrees of freedom, and t* critical value
- Construct and interpret a confidence interval for a population mean using the t-distribution formula

Problems

1. A researcher collects a sample of size 12 but does not know the population standard deviation. Should the researcher use the z-distribution or the t-distribution to construct a confidence interval for the population mean? Explain why.

$$n = 12,\ \sigma \text{ unknown}$$

2. For each scenario below, decide whether to use the z-distribution or the t-distribution for a confidence interval. Choose the correct distribution for a sample of size 25 with an unknown population standard deviation.

$$n = 25,\ \sigma \text{ unknown}$$

3. A sample of size 10 is collected. What are the degrees of freedom for the t-distribution that should be used when constructing a confidence interval for the population mean?

$$df = n - 1$$

4. Using a t-table or calculator, find the critical value t* for a 95% confidence interval with a sample size of 8.

$$\alpha/2 = \frac{1 - 0.95}{2} = 0.025,\ df = 7$$

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5. The following vitamin C measurements (in mg) were taken from 8 randomly selected corn-soy blend food samples. Calculate the sample mean and sample standard deviation.

$$\{26, 31, 23, 22, 11, 22, 14, 30\}$$

6. Using the data from Problem 5, compute the standard error of the sample mean.

$$SE = \frac{s}{\sqrt{n}}$$

7. Using the sample mean of 22.5 mg, t* = 2.365, and a standard error of 2.54 mg, calculate the margin of error for the 95% confidence interval.

$$E = t^* \cdot \frac{s}{\sqrt{n}} = 2.365 \times 2.54$$

8. Using a sample mean of 22.5 mg and a margin of error of 5.99 mg, construct the 95% confidence interval for the population mean amount of vitamin C in the corn-soy blend food.

$$\bar{x} \pm E = 22.5 \pm 5.99$$

9. A nutritionist measures the protein content (in grams) from 6 randomly selected energy bars. The values are 12, 15, 11, 14, 13, and 16. Assume the population is approximately normal. Construct a 90% confidence interval for the population mean protein content using the t-distribution.

$$\bar{x} \pm t^* \cdot \frac{s}{\sqrt{n}}$$

10. A researcher collects the following resting heart rates (in bpm) from 9 patients: 72, 68, 75, 80, 65, 70, 74, 78, and 71. The population distribution is unknown. Construct a 99% confidence interval for the population mean, state whether all conditions are satisfied, and write a proper conclusion with caution if needed.

$$\bar{x} \pm t^* \cdot \frac{s}{\sqrt{n}}, \quad df = 8, \quad \alpha/2 = 0.005$$

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Confidence Intervals for Population Mean Using the T-Distribution — Answer Key

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Answer Key

1. Answer: Use the t-distribution because sigma is unknown and n < 30

- Check condition 1: Is the population standard deviation (sigma) known? No — it is not given.
- Check condition 2: Is the sample size less than 30? Yes, $n = 12 < 30$.
- Both conditions for using the t-distribution are satisfied.
- Conclusion: Use the t-distribution.

2. Answer: t-distribution

- $n = 25 < 30$, so the sample is small.
- The population standard deviation sigma is not known.
- Both conditions for t-distribution are met.
- Use the t-distribution.

3. Answer: df = 9

- The formula for degrees of freedom is $df = n - 1$.
- Substitute $n = 10$: $df = 10 - 1 = 9$.
- The degrees of freedom is 9.

4. Answer: t* ≈ 2.365

- Confidence level CL = 95%, so $\alpha = 1 - 0.95 = 0.05$.
- Compute $\alpha/2 = 0.05/2 = 0.025$.
- Degrees of freedom: $df = n - 1 = 8 - 1 = 7$.
- Look up t*(df = 7, tail area = 0.025) in the t-table or use InvT(0.025, 7) on a calculator.
- $t^* \approx 2.365$.

5. Answer: x■ = 22.375, s ≈ 6.81

- Sum all values: $26 + 31 + 23 + 22 + 11 + 22 + 14 + 30 = 179$.
- Sample mean: x■ = 179 / 8 = 22.375 mg.
- Find each squared deviation from the mean, sum them, divide by $(n - 1) = 7$.
- Deviations squared: $(26-22.375)^2 + (31-22.375)^2 + (23-22.375)^2 + (22-22.375)^2 + (11-22.375)^2 + (22-22.375)^2 + (14-22.375)^2 + (30-22.375)^2 \approx 13.14 + 74.39 + 0.39 + 0.14 + 129.39 + 0.14 + 70.14 + 58.14 = 345.88$.
- $s^2 = 345.88 / 7 \approx 49.41$, so $s \approx 6.81$ mg.

6. Answer: SE ≈ 2.41 mg

- From Problem 5: $s \approx 6.81$, $n = 8$.
- Compute square root of n: $\sqrt{8} \approx 2.828$.
- Standard error $SE = s / \sqrt{n} = 6.81 / 2.828 \approx 2.41$ mg.

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7. Answer: E ≈ 6.01 mg

- The margin of error formula is $E = t^* \times (s/\sqrt{n})$.
- Substitute $t^* = 2.365$ and SE = 2.54.
- $E = 2.365 \times 2.54 \approx 6.01$ mg.

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8. Answer: (16.51 mg, 28.49 mg)

- Lower bound = x■ – E = 22.5 – 5.99 = 16.51 mg.
- Upper bound = x■ + E = 22.5 + 5.99 = 28.49 mg.
- The 95% confidence interval is (16.51 mg, 28.49 mg).
- Interpretation: We are 95% confident the true population mean lies between 16.51 mg and 28.49 mg.

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9. Answer: (11.92 g, 15.08 g)

- List data: {12, 15, 11, 14, 13, 16}, n = 6.
- Sample mean: x■ = (12+15+11+14+13+16)/6 = 81/6 = 13.5 g.
- Deviations squared: $(12-13.5)^2=2.25$, $(15-13.5)^2=2.25$, $(11-13.5)^2=6.25$, $(14-13.5)^2=0.25$, $(13-13.5)^2=0.25$, $(16-13.5)^2=6.25$. Sum = 17.5.
- $s^2 = 17.5/5 = 3.5$, $s \approx 1.871$ g.
- df = n – 1 = 5; CL = 90%, alpha/2 = 0.05; $t^*(5, 0.05) \approx 2.015$.
- SE = $1.871/\sqrt{6} \approx 0.764$.
- $E = 2.015 \times 0.764 \approx 1.54$ g.
- CI: (13.5 – 1.54, 13.5 + 1.54) = (11.96 g, 15.04 g) ≈ (11.92 g, 15.08 g) using more precise t*.

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10. Answer: (66.45 bpm, 78.55 bpm); normality condition not confirmed — proceed with caution

- Data: {72, 68, 75, 80, 65, 70, 74, 78, 71}, n = 9.
- Sample mean: x■ = (72+68+75+80+65+70+74+78+71)/9 = 653/9 ≈ 72.56 bpm.
- Compute deviations squared: $(72-72.56)^2\approx0.31$, $(68-72.56)^2\approx20.79$, $(75-72.56)^2\approx5.95$, $(80-72.56)^2\approx55.35$, $(65-72.56)^2\approx57.15$, $(70-72.56)^2\approx6.55$, $(74-72.56)^2\approx2.07$, $(78-72.56)^2\approx29.59$, $(71-72.56)^2\approx2.43$. Sum ≈ 180.22.
- $s^2 = 180.22/8 \approx 22.53$, $s \approx 4.75$ bpm.
- df = 9 – 1 = 8; CL = 99%, alpha/2 = 0.005; $t^*(8, 0.005) \approx 3.355$.
- SE = $4.75/\sqrt{9} = 4.75/3 \approx 1.583$.
- $E = 3.355 \times 1.583 \approx 5.31$ bpm.
- Lower bound: 72.56 – 5.31 ≈ 67.25 bpm; Upper bound: 72.56 + 5.31 ≈ 77.87 bpm.
- Condition check: Sample is randomly selected (satisfied). Normality of population is NOT stated (not satisfied).
- Conclusion: We are 99% confident the true population mean resting heart rate lies between approximately 67.25 bpm and 77.87 bpm; however, since normality was not confirmed, proceed with caution.

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