Confidence Intervals Using the t-Distribution

Statistics Worksheet · Grade 11–College

Name: _____ Date: _____

Learning Objectives

- Identify the point estimate, margin of error, standard error, and sample standard deviation from a given confidence interval
- Use the t-distribution formula to work backwards from a confidence interval to find unknown statistics
- Interpret a confidence interval in a statistically correct context

Problems

1. A 95% confidence interval for the mean lifetime of AA batteries is (430, 470) minutes. What is the point estimate (sample mean) for this interval?

$(430, 470)$

2. Using the confidence interval (430, 470) minutes, find the margin of error.

$(430, 470)$

3. A 90% confidence interval for a population mean is (118, 132). Find the point estimate and the margin of error.

$(118, 132)$

4. For a t-distribution confidence interval with a 95% confidence level and sample size n = 30, find the tail area used to look up the critical t-value.

$\frac{1 - 0.95}{2}$

5. For a 95% confidence interval with sample size n = 30, determine the degrees of freedom and find the critical t-value (t*) using the inverse-t function.

$df = n - 1$

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6. Given a margin of error of 20, a critical t-value of 2.045, and a sample size of 30, use the margin of error formula to find the sample standard deviation.

$$E = t^* \cdot \frac{s}{\sqrt{n}}$$

7. Using the margin of error of 20 and the critical t-value of approximately 2.045, find the standard error of the mean.

$$SE = \frac{E}{t^*}$$

8. Write a statistically correct interpretation of the 95% confidence interval (430, 470) minutes for the mean lifetime of AA batteries.

9. A 99% confidence interval for the mean weight of a cereal box is (15.6, 16.4) ounces with a sample size of 25. Find the point estimate, margin of error, critical t-value, and sample standard deviation.

$$(15.6,\ 16.4),\ n = 25,\ 99\%$$

10. Two confidence intervals are constructed for the same population mean using the same sample: one at 90% confidence and one at 99% confidence, both with n = 20. The 90% interval is (54.3, 65.7). Without recalculating from raw data, determine the point estimate, find the 90% margin of error, find t* for 90% and 99% (df = 19), calculate the standard deviation, and then construct the 99% confidence interval.

$$(54.3,\ 65.7),\ n = 20,\ 90\% \text{ and } 99\%$$

Confidence Intervals Using the t-Distribution — Answer Key

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Answer Key

1. Answer: 450 minutes

- The point estimate is the midpoint of the confidence interval.
- Point estimate = (430 + 470) / 2 = 900 / 2 = 450

2. Answer: 20 minutes

- The margin of error is the distance from the midpoint to either endpoint.
- Point estimate = 450 (midpoint)
- Margin of error = 470 – 450 = 20 (or equivalently 450 – 430 = 20)

3. Answer: Point estimate = 125; Margin of error = 7

- Point estimate = (118 + 132) / 2 = 250 / 2 = 125
- Margin of error = 132 – 125 = 7

4. Answer: 0.025

- Tail area = (1 – confidence level) / 2
- Tail area = (1 – 0.95) / 2 = 0.05 / 2 = 0.025

5. Answer: df = 29; $t^* \approx 2.045$

- Degrees of freedom = n – 1 = 30 – 1 = 29
- Tail area = 0.025 (from 95% confidence level)
- Using inverse-t: $t^*(0.025, 29) \approx -2.045$; use the positive value $t^* \approx 2.045$

6. Answer: $s \approx 53.57$

- $E = t^* \times (s / \sqrt{n}) \rightarrow 20 = 2.045 \times (s / \sqrt{30})$
- Multiply both sides by $\sqrt{30}$: $20\sqrt{30} = 2.045 \times s$
- $s = (20 \times \sqrt{30}) / 2.045 = (20 \times 5.477) / 2.045 \approx 109.54 / 2.045 \approx 53.57$

7. Answer: SE ≈ 9.78

- The margin of error equals $t^* \times SE$, so $SE = E / t^*$
- $SE = 20 / 2.045 \approx 9.78$

8. Answer: We are 95% confident that the true mean lifetime of AA batteries falls between 430 and 470 minutes.

- A confidence interval interpretation must reference the confidence level, the parameter (true population mean), and the interval bounds.
- Correct wording: 'We are 95% confident that the true mean lifetime of AA batteries is between 430 and 470 minutes.'

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- Avoid saying 'there is a 95% probability the mean is in this interval' — the mean is a fixed (not random) value.

9. Answer: Point estimate = 16 oz; Margin of error = 0.4 oz; t* ≈ 2.797; s ≈ 0.715 oz

- Point estimate = (15.6 + 16.4) / 2 = 16
- Margin of error = 16.4 – 16 = 0.4
- Tail area = (1 – 0.99) / 2 = 0.005; df = 25 – 1 = 24
- Critical value: t*(0.005, 24) ≈ 2.797
- s = E × √n / t* = 0.4 × √25 / 2.797 = 0.4 × 5 / 2.797 = 2 / 2.797 ≈ 0.715

10. Answer: Point estimate = 60; s ≈ 19.44; 99% CI ≈ (47.22, 72.78)

- Point estimate = (54.3 + 65.7) / 2 = 60
- 90% margin of error = 65.7 – 60 = 5.7
- Tail area for 90%: (1 – 0.90)/2 = 0.05; df = 19; t*(0.05, 19) ≈ 1.729
- Find s: s = E × √n / t* = 5.7 × √20 / 1.729 = 5.7 × 4.472 / 1.729 ≈ 25.49 / 1.729 ≈ 14.74
- Tail area for 99%: (1 – 0.99)/2 = 0.005; df = 19; t*(0.005, 19) ≈ 2.861
- 99% margin of error = 2.861 × (14.74 / √20) = 2.861 × 3.296 ≈ 9.43
- 99% CI = (60 – 9.43, 60 + 9.43) ≈ (50.57, 69.43)
- Note: slight variation in final answer is acceptable depending on rounding of t* and s.