

# Confidence Interval for Population Proportion

Statistics Worksheet · Grade 11–12 / AP Statistics

Name: \_\_\_\_\_

Date: \_\_\_\_\_

## Learning Objectives

- Identify the parameter of interest and calculate the sample proportion  $\hat{p}$  for a given scenario
- Verify the three conditions (randomness, independence, normality) required to construct a confidence interval for a population proportion
- Construct and interpret a one-proportion z-interval using the correct formula, critical value, and margin of error

## Problems

1. In a survey, 433 out of 1548 randomly selected US adults felt that unemployment compensation should be extended. Identify the sample proportion  $\hat{p}$ . Round to two decimal places.

$$\hat{p} = \frac{433}{1548}$$

2. Using the sample proportion from Problem 1, find  $\hat{q}$ , the complement of  $\hat{p}$ .

$$\hat{q} = 1 - \hat{p}$$

3. State the three conditions that must be verified before constructing a confidence interval for a population proportion, and describe what each condition checks.

4. Verify the Normality (Large Counts) condition for a sample of 1548 adults where  $\hat{p}$  equals 0.28 and  $\hat{q}$  equals 0.72. Show both calculations.

$$n\hat{p} \geq 10 \quad \text{and} \quad n\hat{q} \geq 10$$

5. Find the critical value  $z^*$  for a 95 percent confidence interval. Show how the tail area is determined before looking up the value.

$$\alpha = 1 - 0.95, \quad \frac{\alpha}{2} = 0.025, \quad z^* = ?$$

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6. Calculate the standard error of the sample proportion using p-hat equal to 0.28, q-hat equal to 0.72, and n equal to 1548. Round to four decimal places.

$$SE = \sqrt{\frac{\hat{p}\hat{q}}{n}} = \sqrt{\frac{0.28 \times 0.72}{1548}}$$

7. Calculate the margin of error for the 95 percent confidence interval using the critical value z-star equal to 1.96 and the standard error equal to 0.0114.

$$ME = z^* \times SE = 1.96 \times 0.0114$$

8. Using p-hat equal to 0.28 and a margin of error of 0.022, construct the 95 percent confidence interval for the population proportion.

$$\hat{p} \pm ME \Rightarrow 0.28 \pm 0.022$$

9. Write a proper interpretation of the 95 percent confidence interval (0.258, 0.302) in the context of this problem about US adults who support extending unemployment compensation.

10. A researcher polls 2000 randomly selected US adults and finds that 580 believe the minimum wage should be raised. All three conditions are met. Construct a 99 percent confidence interval for the true proportion who hold this belief. Show all steps including p-hat, q-hat, z-star, standard error, margin of error, and the final interval.

$$\hat{p} = \frac{580}{2000}, \quad z^* \text{ at } 99\%, \quad CI = \hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

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# Confidence Interval for Population Proportion — Answer Key

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## Answer Key

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### 1. Answer: $\hat{p} \approx 0.28$

- Divide the number of successes by the sample size:  $433 \div 1548$
- $433 \div 1548 \approx 0.2797 \approx 0.28$

### 2. Answer: $\hat{q} = 0.72$

- $\hat{q} = 1 - \hat{p}$
- $\hat{q} = 1 - 0.28 = 0.72$

### 3. Answer: Randomness (sample was randomly selected), Independence (population $\geq 10 \times$ sample size), Normality ( $n\hat{p} \geq 10$ and $n\hat{q} \geq 10$ )

- Randomness: The sample must be randomly selected from the population.
- Independence: The population size must be at least 10 times the sample size (10% condition).
- Normality: Both  $n\hat{p}$  and  $n\hat{q}$  must be at least 10 (Large Counts condition).

### 4. Answer: $n\hat{p} = 433.44 \geq 10$ and $n\hat{q} = 1114.56 \geq 10$ ; condition is satisfied.

- $n\hat{p} = 1548 \times 0.28 = 433.44$ , which is  $\geq 10$ .
- $n\hat{q} = 1548 \times 0.72 = 1114.56$ , which is  $\geq 10$ .
- Both values are at least 10, so the Normality condition is satisfied.

### 5. Answer: $z^* = 1.96$

- $\alpha = 1 - 0.95 = 0.05$
- Each tail area =  $0.05 \div 2 = 0.025$
- Using an inverse normal table or calculator with area 0.025 in the lower tail gives  $z^* = 1.96$

### 6. Answer: $SE \approx 0.0114$

- Numerator:  $0.28 \times 0.72 = 0.2016$
- Divide by  $n$ :  $0.2016 \div 1548 \approx 0.0001302$
- Take the square root:  $\sqrt{0.0001302} \approx 0.0114$

### 7. Answer: $ME \approx 0.022$

- $ME = 1.96 \times 0.0114$
- $ME \approx 0.02234 \approx 0.022$

### 8. Answer: (0.258, 0.302)

- Lower bound:  $0.28 - 0.022 = 0.258$
- Upper bound:  $0.28 + 0.022 = 0.302$
- 95% CI: (0.258, 0.302)

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**9. Answer: We are 95% confident that the true proportion of US adults who feel unemployment compensation should be extended an additional six months is between 0.258 and 0.302.**

- State the confidence level: 95% confident.
- Reference the parameter (true proportion of US adults who support the extension).
- State the interval: between 0.258 and 0.302.

**10. Answer: (0.264, 0.316)**

- $p\text{-hat} = 580 \div 2000 = 0.29$
- $q\text{-hat} = 1 - 0.29 = 0.71$
- For 99% CI:  $\alpha/2 = 0.005$ , so  $z^* = 2.576$
- $SE = \sqrt{(0.29 \times 0.71 \div 2000)} = \sqrt{(0.2059 \div 2000)} = \sqrt{0.00010295} \approx 0.01015$
- $ME = 2.576 \times 0.01015 \approx 0.02615 \approx 0.026$
- Lower bound:  $0.29 - 0.026 = 0.264$
- Upper bound:  $0.29 + 0.026 = 0.316$
- 99% CI: (0.264, 0.316)
- Interpretation: We are 99% confident that the true proportion of US adults who believe the minimum wage should be raised is between 0.264 and 0.316.

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