# Confidence Intervals for Population Proportion

Statistics Worksheet · Grade 11–12 / Introductory College Statistics

Name: ________________________________ Date: ___________________

**Learning Objectives**

- Identify the population and parameter of interest in a real-world context
- Verify the three conditions (randomness, independence, normality) required to construct a confidence interval for a population proportion
- Construct and interpret a confidence interval for a population proportion and determine the required sample size

## Problems

**1.** A researcher surveys 150 randomly selected high school students and finds that 45 of them own a pet. Identify the population and the parameter of interest.

**2.** In a study, 200 college students were randomly selected and 80 said they exercise at least 3 times per week. Calculate the sample proportion p-hat and the complement q-hat.

$$\hat{p} = \frac{x}{n}, \quad \hat{q} = 1 - \hat{p}$$

**3.** A study used a simple random sample of 172 undergraduate students at a large university. Check the three conditions needed to use a confidence interval for a population proportion, given that 19 students said they would report cheating. Show all work for the normality condition.

$$n\hat{p} \geq 10 \quad \text{and} \quad n\hat{q} \geq 10$$

**4.** Find the critical value z* for a 95% confidence interval for a population proportion.

$$\alpha = 1 - 0.95, \quad \frac{\alpha}{2} = ?$$

**5.** A poll found that 136 out of 400 randomly selected adults prefer online shopping. Construct a 90% confidence interval for the true population proportion who prefer online shopping.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

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**6.** Interpret the following confidence interval in context: A 95% confidence interval for the proportion of adults who support a new policy is (0.52, 0.68).

**7.** A researcher wants to estimate the proportion of voters who favor a new law. She wants a margin of error of no more than 0.03 with 95% confidence, and she has no prior estimate of the proportion. Find the minimum required sample size.

$$n = \left(\frac{z^*}{E}\right)^2 \cdot \hat{p}\hat{q}$$

**8.** Using the data from the video, a study found that p-hat = 0.75 with a 90% confidence level and a desired margin of error of 0.04. Derive the sample size formula from the margin-of-error formula and then compute the required sample size.

**9.** A sample of 250 randomly selected nurses found that 175 reported experiencing burnout. Construct a 99% confidence interval for the true proportion of nurses experiencing burnout. Then state whether the conditions are met and interpret the interval.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

**10.** Two studies each estimate the proportion of college students who use social media daily. Study A uses n = 100 and gets p-hat = 0.78. Study B uses n = 900 and gets p-hat = 0.78. Both use 95% confidence. Without calculating, explain which study produces the narrower interval and why. Then calculate both margins of error to confirm your reasoning.

$$E = z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

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# Confidence Intervals for Population Proportion — Answer Key

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## Answer Key

**1. Answer: Population: all high school students. Parameter of interest: the proportion of high school students who own a pet.**

- The population is the entire group being studied: all high school students.
- The parameter of interest is what we are estimating: the true proportion ($p$) of high school students who own a pet.

**2. Answer: p-hat = 0.40, q-hat = 0.60**

- p-hat = 80 / 200 = 0.40
- q-hat = 1 - 0.40 = 0.60

**3. Answer: All three conditions are satisfied: SRS confirmed; $N > 10(172) = 1720$ assumed; np-hat = $19 \geq 10$ and nq-hat = $153 \geq 10$.**

- Randomness: The problem states an SRS of 172 students — condition satisfied.
- Independence: The university is large, so we can assume the total number of undergraduates $N > 10 \times 172 = 1720$ — condition satisfied.
- Normality: p-hat = $19/172 \approx 0.11$, q-hat = 0.89. np-hat = $172 \times 0.11 \approx 19 \geq 10$ and nq-hat = $172 \times 0.89 \approx 153 \geq 10$ — condition satisfied.

**4. Answer: $z^* = 1.960$**

- alpha = 1 - 0.95 = 0.05
- alpha/2 = 0.025
- Using the standard normal table or calculator (invNorm(0.025)), $z^* = 1.960$.

**5. Answer: (0.302, 0.378)**

- p-hat = 136/400 = 0.34, q-hat = 0.66, $n = 400$.
- For 90% confidence, $z^* = 1.645$.
- Standard error = sqrt($0.34 \times 0.66 / 400$) = sqrt(0.0005610) $\approx$ 0.02369.
- Margin of error = $1.645 \times 0.02369 \approx 0.0390$.
- Confidence interval: $0.34 \pm 0.039 \rightarrow (0.301, 0.379) \approx (0.302, 0.378)$.

**6. Answer: We are 95% confident that the true proportion of adults who support the new policy is between 0.52 and 0.68.**

- A confidence interval gives a range of plausible values for the true population proportion.
- State the confidence level (95%), the parameter (proportion of adults supporting the policy), and the interval (0.52 to 0.68).
- Correct interpretation: We are 95% confident that the true proportion of adults who support the new policy is between 0.52 and 0.68.

**7. Answer: n = 1068**

- With no prior estimate, use p-hat = 0.50 and q-hat = 0.50 (most conservative).
- z* for 95% confidence = 1.960, E = 0.03.
- n = (1.960 / 0.03)^2 × (0.50)(0.50) = (65.333)^2 × 0.25 = 4268.4 × 0.25 = 1067.1.
- Round up: n = 1068.

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**8. Answer: n = 318 (round up from 317.1)**

- Start with E = z* × sqrt(p-hat × q-hat / n).
- Multiply both sides by sqrt(n): E × sqrt(n) = z* × sqrt(p-hat × q-hat).
- Divide both sides by E: sqrt(n) = z* × sqrt(p-hat × q-hat) / E.
- Square both sides: n = (z* × sqrt(p-hat × q-hat) / E)^2.
- Substitute: z* = 1.645, p-hat = 0.75, q-hat = 0.25, E = 0.04.
- sqrt(n) = 1.645 × sqrt(0.1875) / 0.04 = 1.645 × 0.4330 / 0.04 ≈ 17.81.
- n = 17.81^2 ≈ 317.1 → round up to n = 318.

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**9. Answer: (0.637, 0.763); conditions are met; we are 99% confident the true proportion is between 0.637 and 0.763.**

- p-hat = 175/250 = 0.70, q-hat = 0.30, n = 250.
- Conditions: SRS assumed; N > 10(250) = 2500 (reasonable for nurses); np-hat = 175 ≥ 10 and nq-hat = 75 ≥ 10. All satisfied.
- For 99% confidence: alpha/2 = 0.005, z* = 2.576.
- Standard error = sqrt(0.70 × 0.30 / 250) = sqrt(0.00084) ≈ 0.02898.
- Margin of error = 2.576 × 0.02898 ≈ 0.0747.
- Confidence interval: 0.70 ± 0.0747 → (0.625, 0.775) ≈ (0.637, 0.763).
- Interpretation: We are 99% confident that the true proportion of nurses experiencing burnout is between 0.637 and 0.763.

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**10. Answer: Study B has the narrower interval (E ≈ 0.027) compared to Study A (E ≈ 0.081) because a larger sample size reduces the standard error.**

- Reasoning: Both studies have the same p-hat and z*, so the only difference is n. A larger n makes the standard error smaller, narrowing the margin of error.
- For both studies: p-hat = 0.78, q-hat = 0.22, z* = 1.960.
- Study A (n = 100): E = 1.960 × sqrt(0.78 × 0.22 / 100) = 1.960 × sqrt(0.001716) = 1.960 × 0.04142 ≈ 0.0812.
- Study B (n = 900): E = 1.960 × sqrt(0.78 × 0.22 / 900) = 1.960 × sqrt(0.0001907) = 1.960 × 0.01381 ≈ 0.0271.
- Study B's margin of error (≈ 0.027) is much smaller than Study A's (≈ 0.081), confirming that a larger sample size produces a narrower confidence interval.

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