Confidence Intervals for Population Proportions

Statistics Worksheet · Grade 11–12 / AP Statistics

Name: ______________________________ Date: __________________

Learning Objectives

- Verify the three conditions (randomness, normality via np > 10 and nq > 10, independence) before constructing a confidence interval for a population proportion
- Construct and interpret a confidence interval for a population proportion using the formula p■ ± z* √(pq/n)
- Determine the minimum sample size required for a given margin of error and confidence level

Problems

1. A survey of 400 randomly selected voters found that 160 plan to vote in the upcoming election. Check whether the normality condition (rule of thumb) is satisfied for constructing a confidence interval for the population proportion.

$$n = 400, \quad \hat{p} = \frac{160}{400} = 0.40, \quad \hat{q} = 0.60$$

2. In a random sample of 250 high school students, 75 reported using social media for more than 5 hours per day. Find the sample proportion p-hat and its complement q-hat.

$$\hat{p} = \frac{x}{n}, \quad \hat{q} = 1 - \hat{p}$$

3. Find the critical value z* for a 90% confidence level using the formula for the critical value.

$$\alpha = 1 - C, \quad \frac{\alpha}{2} = \frac{1 - 0.90}{2}$$

4. A random sample of 500 adults found that 320 regularly exercise at least three times per week. Construct a 95% confidence interval for the true proportion of adults who exercise regularly. Use z* = 1.96.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

5. In a study on smartphone usage, 1,248 out of 4,800 randomly selected teenagers reported experiencing eye strain daily. Check all three conditions for constructing a confidence interval for the population

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proportion, then identify the value of p-hat.

$$\hat{p} = \frac{1248}{4800}, \quad N \geq 10n$$

6. A 99% confidence interval for the proportion of college students who own a laptop is reported as (0.74, 0.86). Identify the sample proportion p-hat and the margin of error from this interval.

$$\hat{p} = \frac{\text{upper} + \text{lower}}{2}, \quad E = \frac{\text{upper} - \text{lower}}{2}$$

7. A health organization wants to estimate the proportion of adults who are physically inactive. They want their estimate to be within 4% at a 95% confidence level. They have no prior estimate of the proportion. Find the minimum sample size needed. Use z* = 1.96 and p-hat = 0.50.

$$n \geq (z^*)^2 \cdot \frac{\hat{p}\hat{q}}{E^2}$$

8. A university poll of 2,200 randomly chosen undergraduates found that 594 had taken an online-only course in the past year. Construct a 99% confidence interval for the true proportion of undergraduates who have taken an online-only course. Use z* = 2.576.

$$\hat{p} \pm 2.576\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

9. A market research firm previously found that 35% of consumers preferred a particular brand. They want to update this estimate to within 2% at a 90% confidence level. Use z* = 1.645 and p-hat = 0.35. What is the minimum sample size needed? How does this compare to the sample size required if no prior estimate were available (p-hat = 0.50)?

$$n \geq \frac{(z^*)^2\hat{p}\hat{q}}{E^2}$$

10. A national polling agency surveyed 8,500 randomly selected registered voters and found that 3,740 supported a proposed education reform bill. (a) Verify all three conditions for a confidence interval for the population proportion. (b) Construct a 95% confidence interval using z* = 1.96. (c) Write a conclusion interpreting the interval in context. (d) If the agency wants to conduct a follow-up survey accurate to within 1.5% at the 99% confidence level using the proportion found here, what is the minimum sample size needed? Use z* = 2.576.

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$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}, \quad n \geq \frac{(z^*)^2 \hat{p}\hat{q}}{E^2}$$

Confidence Intervals for Population Proportions — Answer Key

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Answer Key

1. Answer: np = 160 > 10 and nq = 240 > 10; normality condition is satisfied

- Calculate p■ = 160/400 = 0.40 and q■ = 1 – 0.40 = 0.60
- Check np■ = 400 × 0.40 = 160 > 10 ✓
- Check nq■ = 400 × 0.60 = 240 > 10 ✓
- Both conditions are satisfied, so the normality condition holds.

2. Answer: p■ = 0.30, q■ = 0.70

- p■ = 75 / 250 = 0.30
- q■ = 1 – 0.30 = 0.70

3. Answer: z* = 1.645

- α = 1 – 0.90 = 0.10
- α/2 = 0.05
- Look up the z-value that leaves 0.05 in each tail: z* = 1.645

4. Answer: (0.598, 0.682)

- p■ = 320/500 = 0.64, q■ = 0.36
- Standard error = √(0.64 × 0.36 / 500) = √(0.0004608) ≈ 0.02147
- Margin of error = 1.96 × 0.02147 ≈ 0.0421
- Lower bound = 0.64 – 0.0421 = 0.5979 ≈ 0.598
- Upper bound = 0.64 + 0.0421 = 0.6821 ≈ 0.682
- 95% CI: (0.598, 0.682)

5. Answer: p■ = 0.26; all three conditions satisfied (random ✓, np = 1248 > 10 ✓, nq = 3552 > 10 ✓, N ≥ 48,000 ✓)

- Condition 1 – Randomness: stated as randomly selected ✓
- p■ = 1248 / 4800 = 0.26, q■ = 0.74
- Condition 2 – Normality: np■ = 1248 > 10 ✓ and nq■ = 3552 > 10 ✓
- Condition 3 – Independence: The total number of teenagers N must be ≥ 10 × 4800 = 48,000; this is very likely ✓
- p■ = 0.26

6. Answer: p■ = 0.80, E = 0.06

- p■ = (0.74 + 0.86) / 2 = 1.60 / 2 = 0.80
- E = (0.86 – 0.74) / 2 = 0.12 / 2 = 0.06

7. Answer: n ≥ 601

- Use p■ = q■ = 0.50 (worst-case scenario when no prior estimate is available)

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- E = 0.04, z* = 1.96
- n ≥ $(1.96)^2$ × (0.50 × 0.50) / $(0.04)^2$
- n ≥ 3.8416 × 0.25 / 0.0016
- n ≥ 0.9604 / 0.0016 = 600.25
- Round up: n ≥ 601

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8. Answer: (0.244, 0.296)

- p■ = 594 / 2200 = 0.27, q■ = 0.73
- Standard error = √(0.27 × 0.73 / 2200) = √(0.00008959) ≈ 0.009465
- Margin of error = 2.576 × 0.009465 ≈ 0.02438
- Lower bound = 0.27 – 0.02438 ≈ 0.2456 ≈ 0.246
- Upper bound = 0.27 + 0.02438 ≈ 0.2944 ≈ 0.294
- 99% CI: approximately (0.244, 0.296)

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9. Answer: n ≥ 1,537 using p■ = 0.35; n ≥ 1,692 using p■ = 0.50; prior estimate reduces required sample size

- Using p■ = 0.35, q■ = 0.65, z* = 1.645, E = 0.02:
- n ≥ $(1.645)^2$ × (0.35 × 0.65) / $(0.02)^2$
- n ≥ 2.706025 × 0.2275 / 0.0004 = 0.615620 / 0.0004 = 1539.05 → n ≥ 1,540
- Using p■ = 0.50, q■ = 0.50:
- n ≥ 2.706025 × 0.25 / 0.0004 = 0.676506 / 0.0004 = 1691.27 → n ≥ 1,692
- Using the prior estimate of p■ = 0.35 requires a smaller sample (1,540 vs 1,692).

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10. Answer: (a) All 3 conditions satisfied; (b) 95% CI: (0.430, 0.451); (c) We are 95% confident the true proportion of registered voters supporting the bill is between 43.0% and 45.1%; (d) n ≥ 7,212

- (a) Condition 1: Randomly selected ✓; p■ = 3740/8500 = 0.44, q■ = 0.56
- Condition 2: np■ = 3740 > 10 ✓ and nq■ = 4760 > 10 ✓
- Condition 3: Total registered voters N >> 10 × 8500 = 85,000 ✓
- (b) SE = √(0.44 × 0.56 / 8500) = √(0.00002898) ≈ 0.005384
- E = 1.96 × 0.005384 ≈ 0.01055
- Lower = 0.44 – 0.01055 ≈ 0.4295; Upper = 0.44 + 0.01055 ≈ 0.4505
- 95% CI: (0.430, 0.451)
- (c) We are 95% confident the true proportion of registered voters who support the education reform bill is between 43.0% and 45.1%.
- (d) n ≥ $(2.576)^2$ × (0.44 × 0.56) / $(0.015)^2$
- n ≥ 6.635776 × 0.2464 / 0.000225 = 1.634833 / 0.000225 ≈ 7,266 → n ≥ 7,266

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