Determining Sample Size for Estimating a Population Mean

Statistics Worksheet · Grade 11–12 / Introductory College Statistics

Name: ________________________________ Date: ___________________

Learning Objectives

- Apply the sample size formula $n \geq (z^* \cdot \sigma / E)^2$ to find the minimum sample size needed for a given margin of error and confidence level.
- Identify and interpret the components of the margin of error formula: critical value (z^*), population standard deviation (σ), and maximum error of estimate (E).
- Analyze real-world scenarios to determine how changes in confidence level, margin of error, and standard deviation affect the required sample size.

Problems

1. A researcher wants to estimate a population mean with a maximum error of estimate of 3, a population standard deviation of 12, and a 95% confidence level ($z^* = 1.96$). Use the sample size formula to find the minimum sample size required. Round up to the nearest whole number.

$$n \geq \left(\frac{z^* \cdot \sigma}{E}\right)^2$$

2. A study uses a 90% confidence level, which has a critical value of $z^* = 1.645$. The population standard deviation is 20, and the desired maximum error of estimate is 5. What is the minimum sample size needed?

$$n \geq \left(\frac{z^* \cdot \sigma}{E}\right)^2$$

3. A nutritionist wants to estimate the mean daily caloric intake of adults. A previous study showed the population standard deviation is 250 calories. She wants the estimate to be within 50 calories at a 95% confidence level ($z^* = 1.96$). How many adults must she sample?

$$n \geq \left(\frac{1.96 \cdot 250}{50}\right)^2$$

4. An environmental scientist wants to estimate the mean level of a pollutant in a river. The population standard deviation is 8 parts per million (ppm). She wants a maximum error of 2 ppm at a 99% confidence level ($z^* = 2.576$). What is the minimum sample size?

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$$n \geq \left(\frac{2.576 \cdot 8}{2}\right)^2$$

5. The table below shows four research scenarios with different values of z*, σ, and E. Fill in the missing minimum sample sizes by applying the sample size formula. Round all answers up to the nearest whole number.

Scenario	z*	σ	E	Minimum n
A	1.96	15	3	
B	1.645	10	2	
C	2.576	25	5	
D	1.96	40	8	

6. A hospital administrator wants to estimate the mean waiting time in the emergency room. The population standard deviation from past records is 18 minutes. She wants a 95% confidence interval (z* = 1.96) with a maximum error of 4 minutes. Determine the minimum number of patients that must be included in the sample.

$$n \geq \left(\frac{1.96 \cdot 18}{4}\right)^2$$

7. A researcher is comparing two sampling plans. Plan A uses a 95% confidence level (z* = 1.96) with a maximum error of 5, and Plan B uses a 99% confidence level (z* = 2.576) with the same maximum error of 5. Both plans have a population standard deviation of 30. Find the minimum sample size for each plan, and explain how the change in confidence level affects the required sample size.

$$n \geq \left(\frac{z^* \cdot \sigma}{E}\right)^2$$

8. A biologist wants to estimate the mean length of a species of fish. The population standard deviation is known to be 3.5 cm. She initially plans a 95% confidence interval (z* = 1.96) with a maximum error of 1 cm. Midway through planning, she decides to halve the maximum error to 0.5 cm. By what factor does the required minimum sample size change, and what is the new sample size?

$$n \geq \left(\frac{z^* \cdot \sigma}{E}\right)^2$$

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9. A public health researcher wants to estimate the mean blood pressure of adults in a city at a 95% confidence level (z* = 1.96). She can afford to sample at most 150 people. Previous records show the population standard deviation is 14 mmHg. What is the minimum maximum error of estimate she can achieve with exactly 150 participants? Round to two decimal places.

$$E = z^* \cdot \frac{\sigma}{\sqrt{n}}$$

10. A pharmaceutical company is conducting a clinical trial to estimate the mean reduction in LDL cholesterol (in mg/dL) produced by a new drug. From pilot studies, the population standard deviation is estimated to be 22 mg/dL. The company wants to be 99% confident (z* = 2.576) that the estimate is within 6 mg/dL of the true mean. Due to budget constraints, the maximum allowable sample size is 100 patients. Determine the minimum required sample size. Does it exceed the budget constraint? If so, what maximum error of estimate can the company achieve with exactly 100 patients at the 99% confidence level?

$$n \geq \left(\frac{z^* \cdot \sigma}{E}\right)^2, \quad E = z^* \cdot \frac{\sigma}{\sqrt{n}}$$

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Determining Sample Size for Estimating a Population Mean — Answer Key

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Answer Key

1. Answer: n = 62

- Identify the values: $z^* = 1.96$, $\sigma = 12$, $E = 3$
- Substitute into the formula: $n \geq (1.96 \times 12 / 3)^2$
- Simplify inside the parentheses: $1.96 \times 12 = 23.52$; $23.52 / 3 = 7.84$
- Square the result: $7.84^2 = 61.47$
- Round up to the next whole number: $n = 62$

2. Answer: n = 44

- Identify the values: $z^* = 1.645$, $\sigma = 20$, $E = 5$
- Substitute: $n \geq (1.645 \times 20 / 5)^2$
- Simplify: $1.645 \times 20 = 32.9$; $32.9 / 5 = 6.58$
- Square: $6.58^2 = 43.30$
- Round up: $n = 44$

3. Answer: n = 97

- Identify: $z^* = 1.96$, $\sigma = 250$, $E = 50$
- Calculate numerator: $1.96 \times 250 = 490$
- Divide by E: $490 / 50 = 9.8$
- Square: $9.8^2 = 96.04$
- Round up: $n = 97$

4. Answer: n = 107

- Identify: $z^* = 2.576$, $\sigma = 8$, $E = 2$
- Calculate numerator: $2.576 \times 8 = 20.608$
- Divide by E: $20.608 / 2 = 10.304$
- Square: $10.304^2 = 106.17$
- Round up: $n = 107$

5. Answer: A: 97, B: 68, C: 166, D: 97

Scenario	z*	σ	E	Minimum n
A	1.96	15	3	97
B	1.645	10	2	68
C	2.576	25	5	166

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Scenario	z*	σ	E	Minimum n
D	1.96	40	8	97

- A: $(1.96 \times 15 / 3)^2 = (9.8)^2 = 96.04 \to n = 97$
- B: $(1.645 \times 10 / 2)^2 = (8.225)^2 = 67.65 \to n = 68$
- C: $(2.576 \times 25 / 5)^2 = (12.88)^2 = 165.89 \to n = 166$
- D: $(1.96 \times 40 / 8)^2 = (9.8)^2 = 96.04 \to n = 97$

6. Answer: n = 78

- Identify: $z^* = 1.96$, $\sigma = 18$, $E = 4$
- Calculate numerator: $1.96 \times 18 = 35.28$
- Divide by E: $35.28 / 4 = 8.82$
- Square: $8.82^2 = 77.79$
- Round up: $n = 78$

7. Answer: Plan A: n = 139; Plan B: n = 240; higher confidence level requires a larger sample size.

- Plan A: $(1.96 \times 30 / 5)^2 = (11.76)^2 = 138.30 \to n = 139$
- Plan B: $(2.576 \times 30 / 5)^2 = (15.456)^2 = 238.89 \to n = 240$
- Increasing the confidence level increases z*, which increases the numerator, resulting in a larger required sample size.
- Conclusion: A 99% confidence level requires a notably larger sample than a 95% confidence level when all other factors remain the same.

8. Answer: Original n = 47; New n = 188; sample size increases by a factor of 4.

- Original: $n \geq (1.96 \times 3.5 / 1)^2 = (6.86)^2 = 47.06 \to n = 47$
- New: $n \geq (1.96 \times 3.5 / 0.5)^2 = (13.72)^2 = 188.24 \to n = 189$
- Wait — recalculate carefully: $1.96 \times 3.5 = 6.86$; $6.86 / 0.5 = 13.72$; $13.72^2 = 188.24 \to n = 189$
- Ratio: $189 / 47 \approx 4$; halving E quadruples the required sample size (since n is proportional to $1/E^2$).
- Key insight: The sample size is inversely proportional to E^2, so cutting E in half multiplies n by 4.

9. Answer: E ≈ 2.24 mmHg

- Rearrange the margin of error formula to solve for E: $E = z^* \times (\sigma / \sqrt{n})$
- Substitute: $E = 1.96 \times (14 / \sqrt{150})$
- Calculate $\sqrt{150} \approx 12.247$
- Divide: $14 / 12.247 \approx 1.1431$
- Multiply: $1.96 \times 1.1431 \approx 2.24$
- The minimum achievable maximum error with n = 150 is approximately 2.24 mmHg.

10. Answer: Required n = 90, which is within the budget. The achieved E with n = 100 is approximately 5.67 mg/dL.

- Step 1 — Find minimum n: $n \geq (2.576 \times 22 / 6)^2$
- Numerator: $2.576 \times 22 = 56.672$
- Divide: $56.672 / 6 = 9.445$
- Square: $9.445^2 = 89.21 \to$ round up to n = 90
- Step 2 — Check budget: $90 \leq 100$, so the required sample size is within the budget.
- Step 3 — Find E with n = 100: $E = 2.576 \times (22 / \sqrt{100}) = 2.576 \times (22 / 10) = 2.576 \times 2.2 = 5.667 \approx 5.67$ mg/dL

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• Conclusion: The company can meet the 6 mg/dL error goal with n = 90 patients. Using all 100 patients reduces the maximum error to approximately 5.67 mg/dL, improving precision.

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