# Hypothesis Testing for Population Mean

Statistics Worksheet · Grade 11–12 / Introductory College Statistics

Name: ________________________________ Date: ___________________

**Learning Objectives**

- State null and alternative hypotheses for a population mean using correct notation
- Calculate a z-test statistic and find the corresponding P-value
- Make a conclusion about the null hypothesis by comparing the P-value to a given significance level

## Problems

**1.** A teacher claims the mean score on a standardized math test is 75. Write the null hypothesis and the alternative hypothesis if a researcher believes the true mean is different from 75. Use mu to represent the population mean.

$$H_0 : \mu = 75 \quad \text{vs.} \quad H_a : \mu \neq 75$$

**2.** A nutritionist believes that adults consume MORE than 2000 calories per day on average. Write the null and alternative hypotheses for this claim using mu.

$$H_0 : \mu = 2000 \quad \text{vs.} \quad H_a : \mu > 2000$$

**3.** A researcher wants to test whether the mean resting heart rate for athletes is less than 65 beats per minute. State both hypotheses and identify the type of test (left-tailed, right-tailed, or two-tailed).

$$H_0 : \mu = 65 \quad \text{vs.} \quad H_a : \mu < 65$$

**4.** Before conducting a hypothesis test for a population mean, you must check three conditions. A researcher randomly selects 40 students from a large university, and the population of scores is stated to be normally distributed. The sample size is less than 10% of the total student population. State whether all three conditions are satisfied and identify each condition.

**5.** The population mean weight of a cereal box is claimed to be 400 grams with a known population standard deviation of 10 grams. A quality inspector randomly selects 25 boxes and finds a sample mean of 396 grams. Calculate the z-test statistic.

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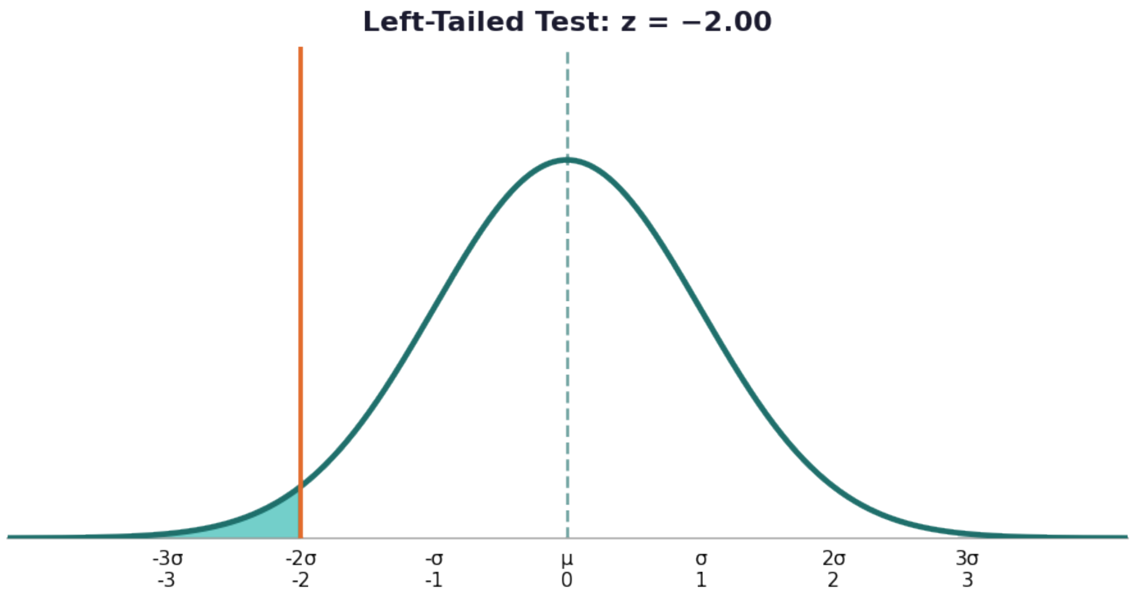
$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

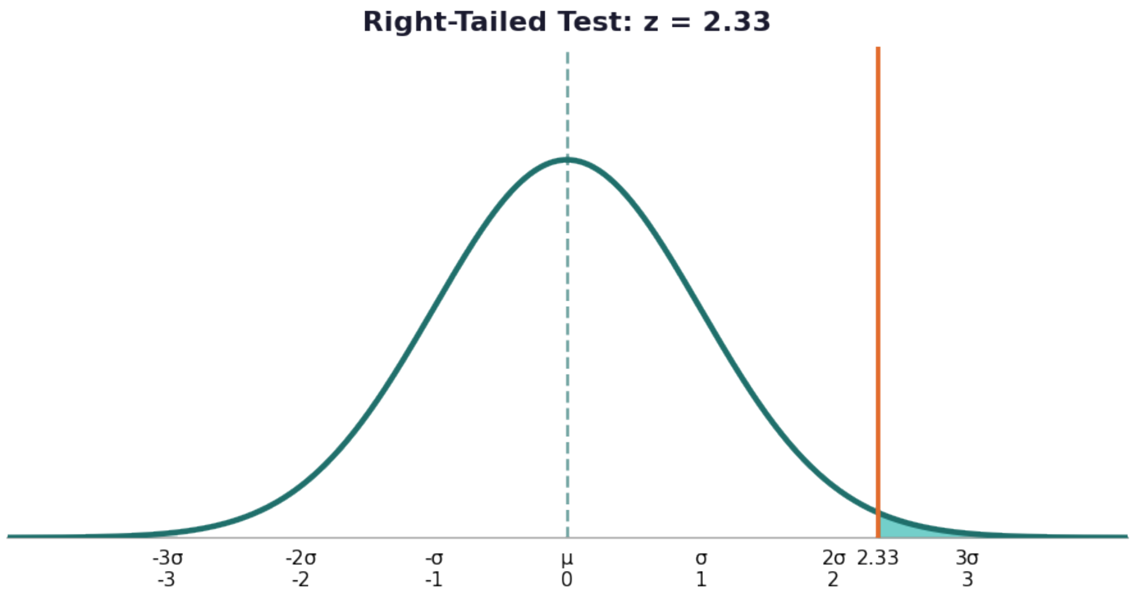
**6.** Using the z-test statistic of negative 2.00 from the cereal box problem above, find the P-value for a left-tailed test. The shaded area below shows the region of interest. Then state whether you reject or fail to reject the null hypothesis at a significance level of 0.05.




**7.** The mean score for US college students on the Survey of Study Habits and Attitudes (SSHA) is 115 with a known population standard deviation of 30. Monica suspects that older students (age 30 and above) have better attitudes toward school. She randomly selects 25 such students and finds a sample mean score of 118.6. At a significance level of 0.05, carry out all five steps of the hypothesis test.

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

**8.** A hospital claims the average patient wait time is 30 minutes. A health inspector believes the actual wait time is longer. She samples 36 patients and finds a mean wait time of 33.5 minutes. The population standard deviation is known to be 9 minutes. Conduct a full hypothesis test at alpha equals 0.01.




**9.** The table below summarizes three hypothesis tests. For each test, determine whether to reject or fail to reject the null hypothesis based on the given P-value and significance level. Fill in the Decision and Conclusion columns.

| Test | P-value | Alpha (α) | Decision | Conclusion |
|---|---|---|---|---|
| A | 0.032 | 0.05 | | |
| B | 0.078 | 0.05 | | |
| C | 0.004 | 0.01 | | |

**10.** A manufacturer claims that the mean lifetime of their light bulbs is 1500 hours with a known population standard deviation of 120 hours. A consumer group believes the mean lifetime is actually different from 1500 hours. They randomly test 64 bulbs and find a sample mean lifetime of 1468 hours. The population is normally distributed. Perform a complete hypothesis test at alpha equals 0.05, compute the z-statistic and P-value, and write a full conclusion in the context of the problem.

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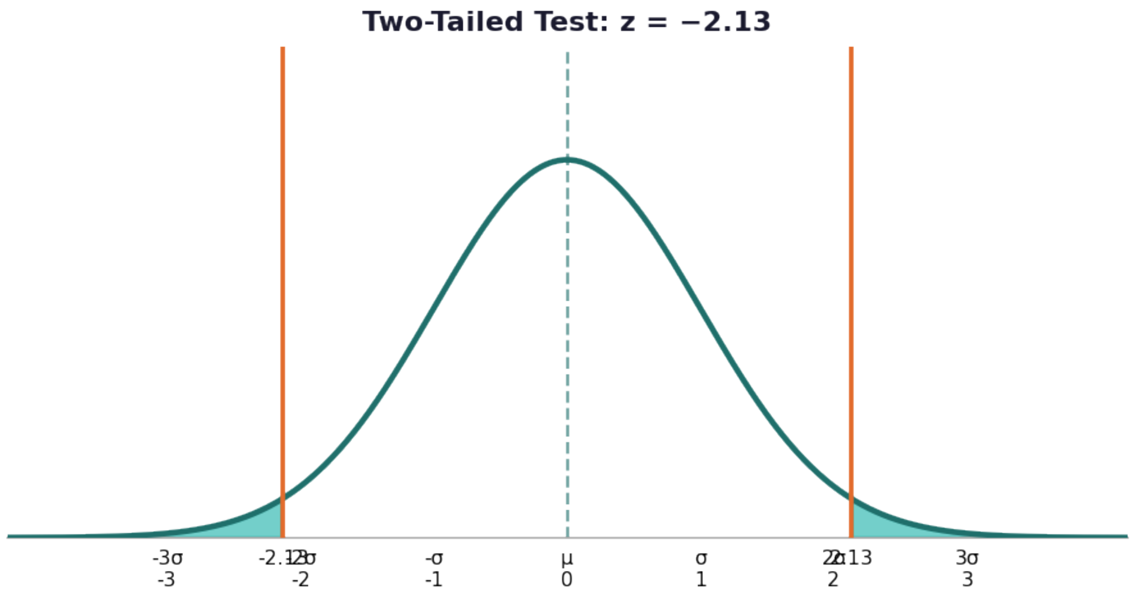
Two-Tailed Test: z = −2.13
-3σ
-2.13
-2σ
-σ
μ
σ
2σ
2.13
3σ
-3
-2
-1
0
1
2
3

# Hypothesis Testing for Population Mean — Answer Key

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## Answer Key

**1. Answer: H■: μ = 75; H■: μ ≠ 75 (two-tailed test)**

- The null hypothesis always states equality: H■: μ = 75.
- Because the researcher believes the mean is DIFFERENT (not specifically higher or lower), the alternative is two-tailed: H■: μ ≠ 75.

**2. Answer: H■: μ = 2000; H■: μ > 2000 (right-tailed test)**

- The null hypothesis is: H■: μ = 2000 (no change from the assumed value).
- Because the claim is that adults consume MORE than 2000, the alternative is right-tailed: H■: μ > 2000.

**3. Answer: H■: μ = 65; H■: μ < 65 — Left-tailed test**

- The null hypothesis states the mean equals the claimed value: H■: μ = 65.
- Because the researcher suspects the mean is LESS than 65, the alternative is left-tailed: H■: μ < 65.
- This is a left-tailed test because the rejection region is in the left tail.

**4. Answer: All three conditions are satisfied: (1) Random sample ✓, (2) Normal population ✓, (3) Independence (n < 10% of population) ✓**

- Condition 1 — Random: The problem states the sample was randomly selected. ✓
- Condition 2 — Normality: The population is stated to be normally distributed. ✓
- Condition 3 — Independence: The sample of 40 is less than 10% of the large university population. ✓
- Since all three conditions are met, we may proceed with hypothesis testing without caution.

**5. Answer: z = –2.00**

- Identify values: x■ = 396, μ■ = 400, σ = 10, n = 25.
- Calculate standard error: σ / √n = 10 / √25 = 10 / 5 = 2.
- Calculate z: z = (396 – 400) / 2 = –4 / 2 = –2.00.
- The z-test statistic is –2.00, indicating the sample mean is 2 standard errors below the claimed mean.

**6. Answer: P-value ≈ 0.0228; Since 0.0228 < 0.05, reject H■.**

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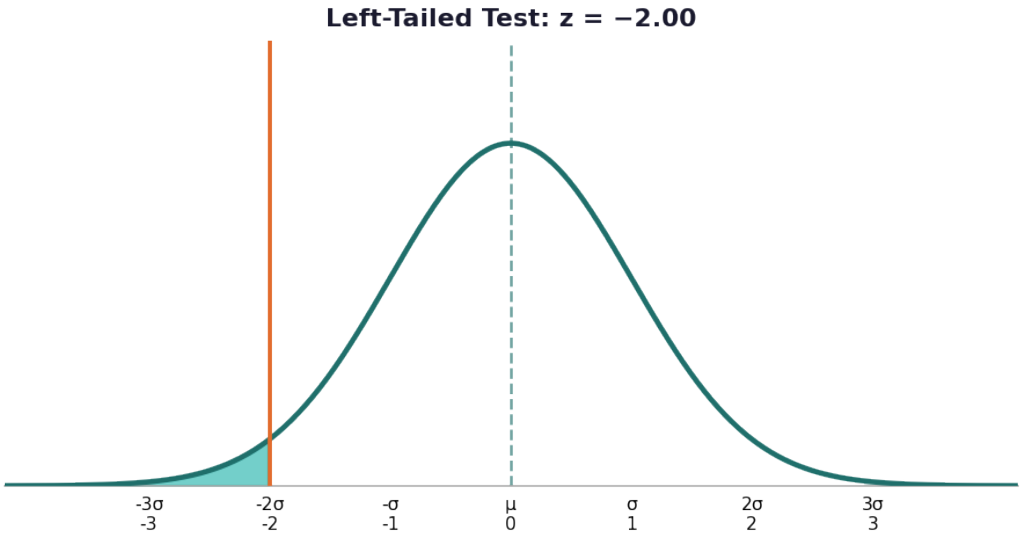


- For a left-tailed test with z = –2.00, find P(Z < –2.00) using the standard normal table.
- P(Z < –2.00) ≈ 0.0228.
- Compare: P-value = 0.0228 < α = 0.05.
- Decision: Since the P-value is less than α, reject the null hypothesis.
- Conclusion: There is sufficient evidence that the mean cereal box weight is less than 400 grams.

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**7. Answer: z ≈ 0.60; P-value ≈ 0.2743; Fail to reject H■. Insufficient evidence that older students score higher.**

- Step 1 — Hypotheses: H■: μ = 115; H■: μ > 115 (right-tailed, Monica suspects higher scores).
- Step 2 — Conditions: Random sample ✓; Population normally distributed ✓; n = 25 < 10% of all students ✓.
- Step 3 — Test statistic: z = (118.6 – 115) / (30 / √25) = 3.6 / 6 = 0.60.
- Step 4 — P-value: P(Z > 0.60) = 1 – 0.7257 = 0.2743.
- Step 5 — Conclusion: 0.2743 > 0.05, so we fail to reject H■. There is not sufficient evidence that older students have a higher mean SSHA score than the general population.

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**8. Answer: z ≈ 2.33; P-value ≈ 0.0099; Reject H■ at α = 0.01. Evidence supports longer wait times.**

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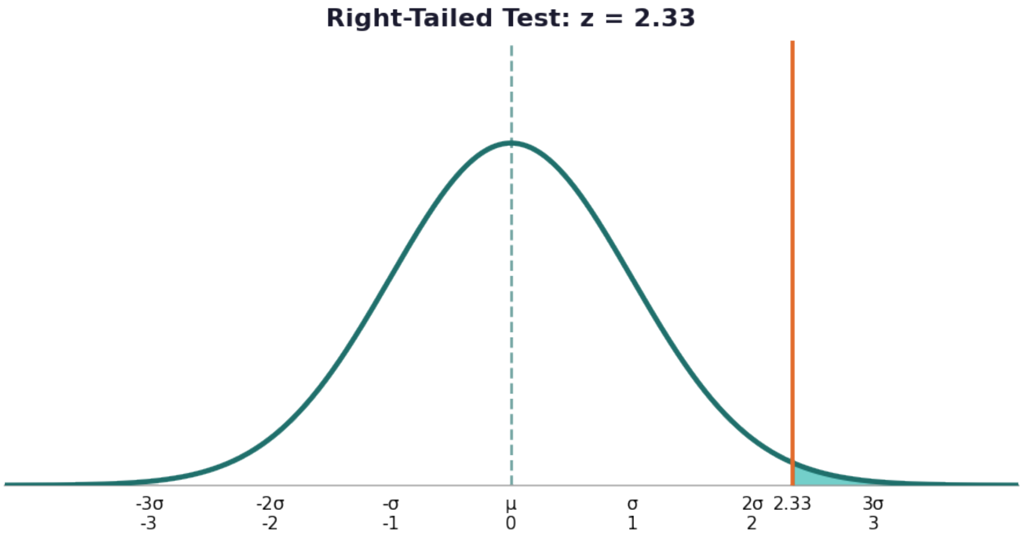


- Step 1 — Hypotheses: H■: μ = 30; H■: μ > 30 (right-tailed).
- Step 2 — Conditions: Random sample ✓; Assume population is normally distributed ✓; n = 36 < 10% of all patients ✓.
- Step 3 — Test statistic: z = (33.5 – 30) / (9 / √36) = 3.5 / 1.5 ≈ 2.33.
- Step 4 — P-value: P(Z > 2.33) = 1 – 0.9901 = 0.0099.
- Step 5 — Conclusion: 0.0099 < 0.01, so we reject H■. There is sufficient evidence at the 0.01 level that the mean patient wait time exceeds 30 minutes.

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**9. Answer: Test A: Reject H■; Test B: Fail to reject H■; Test C: Reject H■**

| Test | P-value | Alpha (α) | Decision | Conclusion |
|---|---|---|---|---|
| A | 0.032 | 0.05 | Reject $H_0$ | Sufficient evidence against $H_0$ |
| B | 0.078 | 0.05 | Fail to reject $H_0$ | Insufficient evidence against $H_0$ |
| C | 0.004 | 0.01 | Reject $H_0$ | Sufficient evidence against $H_0$ |

- Rule: If P-value ≤ α, reject H■. If P-value > α, fail to reject H■.
- Test A: 0.032 ≤ 0.05 → Reject H■.
- Test B: 0.078 > 0.05 → Fail to reject H■.
- Test C: 0.004 ≤ 0.01 → Reject H■.

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**10. Answer: z ≈ –2.13; P-value ≈ 0.0332; Reject H■. Evidence that mean bulb lifetime differs from 1500 hours.**

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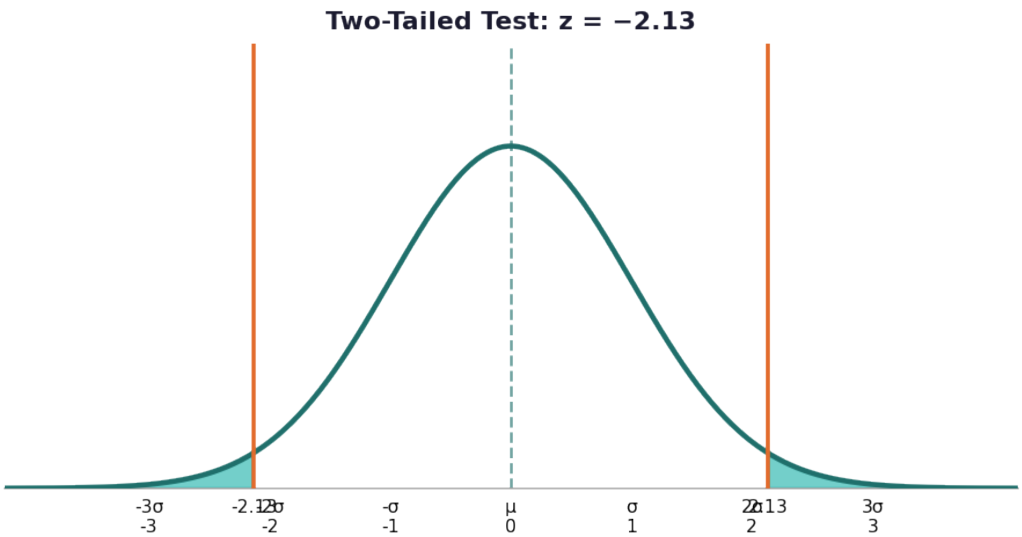


- Step 1 — Hypotheses: H■: μ = 1500; H■: μ ≠ 1500 (two-tailed, consumer group believes it's different).
- Step 2 — Conditions: Random sample ✓; Population normally distributed ✓; n = 64 < 10% of all bulbs produced ✓. All conditions met.
- Step 3 — Test statistic: SE = 120 / √64 = 120 / 8 = 15. z = (1468 – 1500) / 15 = –32 / 15 ≈ –2.13.
- Step 4 — P-value (two-tailed): P = 2 × P(Z < –2.13) = 2 × 0.0166 = 0.0332.
- Step 5 — Conclusion: Since P-value = 0.0332 < α = 0.05, we reject H■. At the 5% significance level, there is sufficient evidence to conclude that the mean lifetime of the manufacturer's light bulbs is significantly different from 1500 hours.