Hypothesis Testing for One Sample Proportion

Statistics Worksheet · Grade 11–12 / Intro College Stats

Name: ________________________________ Date: ____________________

Learning Objectives

- Write null and alternative hypotheses for a one-sample proportion test
- Verify conditions (normality, independence, randomness) before conducting a test
- Calculate the z-test statistic and use the p-value to draw a conclusion

Problems

1. A company claims that 60% of its customers are satisfied with their service. Identify the null and alternative hypotheses if a researcher wants to test whether the true proportion differs from 60%.

$$H_0 : p = 0.60 \quad H_a : p \neq 0.60$$

2. A politician claims that more than 50% of voters support her. Write the null and alternative hypotheses for this claim.

$$H_0 : p = 0.50 \quad H_a : p > 0.50$$

3. A sample of 80 students is drawn. The hypothesized population proportion is p = 0.40. Check whether the normality condition is satisfied using the rule: n·p > 10 and n·q > 10.

$$n = 80, \quad p = 0.40, \quad q = 1 - 0.40$$

4. A survey of 50 people finds that 18 prefer brand A. Calculate the sample proportion (p-hat).

$$\hat{p} = \frac{x}{n} = \frac{18}{50}$$

5. Using a sample of n = 100 with p-hat = 0.55 and a hypothesized population proportion of p = 0.50, calculate the z-test statistic.

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

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6. Ryan, a basketball player, made 40% of his free throws last season. This season he made 25 out of 40 attempts. Test at the 1% significance level whether his proportion has improved. First, write the hypotheses and check the normality condition.

$$H_0 : p = 0.40 \quad H_a : p > 0.40$$

7. Continuing the Ryan example: p = 0.40, n = 40, and he made 25 out of 40 free throws. Calculate the z-test statistic.

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{\frac{25}{40} - 0.40}{\sqrt{\frac{(0.40)(0.60)}{40}}}$$

8. Using the z-test statistic of approximately 3.23 from the Ryan free throw problem and a right-tailed test at the 1% significance level (alpha = 0.01, critical z = 2.326), state the conclusion. The p-value for z = 3.23 is approximately 0.0006.

$$\alpha = 0.01, \quad z = 3.23, \quad p\text{-value} \approx 0.0006$$

9. A health agency claims that less than 25% of adults in a city exercise regularly. A random sample of 200 adults finds that 44 exercise regularly. At the 5% significance level, test the agency's claim. Show all four steps: (1) hypotheses, (2) conditions, (3) test statistic, (4) conclusion. The p-value for this test is approximately 0.1335.

$$H_0 : p = 0.25 \quad H_a : p < 0.25$$

10. A manufacturer claims that exactly 90% of its light bulbs last more than 1,000 hours. A quality inspector randomly tests 150 bulbs and finds that 126 last more than 1,000 hours. At the 5% significance level, test whether the true proportion differs from 90%. The p-value for this test is approximately 0.0241. State all four steps, identify the type of test, and write a full conclusion in context.

$$H_0 : p = 0.90 \quad H_a : p \neq 0.90$$

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Hypothesis Testing for One Sample Proportion — Answer Key

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Answer Key

1. Answer: H■: p = 0.60 (two-tailed test)

- The null hypothesis always uses an equals sign: H■: p = 0.60
- Because the researcher tests whether the proportion 'differs' (not simply higher or lower), the alternative is two-tailed: H■: $p \neq 0.60$

2. Answer: H■: p = 0.50, H■: p > 0.50 (right-tailed test)

- The null hypothesis states the proportion equals the claimed value: H■: p = 0.50
- Because the claim is 'more than,' the alternative hypothesis is right-tailed: H■: $p > 0.50$

3. Answer: Normality condition is satisfied: np = 32 > 10 and nq = 48 > 10

- Compute $q = 1 - 0.40 = 0.60$
- $n \cdot p = 80 \times 0.40 = 32 > 10$ ✓
- $n \cdot q = 80 \times 0.60 = 48 > 10$ ✓
- Both conditions are met, so normality is satisfied.

4. Answer: p-hat = 0.36

- Divide the number of successes by the sample size: p■ = 18 ÷ 50
- p■ = 0.36

5. Answer: z ≈ 1.00

- Identify values: p■ = 0.55, p = 0.50, q = 0.50, n = 100
- Compute standard deviation: $\sqrt{(0.50 \times 0.50 / 100)} = \sqrt{(0.0025)} = 0.05$
- $z = (0.55 - 0.50) / 0.05 = 0.05 / 0.05 = 1.00$

6. Answer: H■: p > 0.40 (right-tailed); np = 16 > 10, nq = 24 > 10 — normality satisfied

- The parameter p = proportion of Ryan's successful free throws
- H■: p = 0.40 (no improvement); H■: p > 0.40 (improvement) — right-tailed
- Check normality: $n \cdot p = 40 \times 0.40 = 16 > 10$ ✓ and $n \cdot q = 40 \times 0.60 = 24 > 10$ ✓
- Normality condition is satisfied.

7. Answer: z ≈ 3.23

- p■ = 25/40 = 0.625
- Standard deviation = $\sqrt{(0.40 \times 0.60 / 40)} = \sqrt{(0.24/40)} = \sqrt{0.006} \approx 0.07746$
- $z = (0.625 - 0.40) / 0.07746 = 0.225 / 0.07746 \approx 3.23$

8. Answer: Reject H■. There is sufficient evidence that Ryan's free throw proportion has improved.

- Compare the p-value to α: $0.0006 < 0.01$

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- Since p-value < α, we reject the null hypothesis.
- Conclusion: At the 1% significance level, there is sufficient evidence to conclude that Ryan's free throw proportion has increased above 40%.

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9. Answer: Fail to reject H■; insufficient evidence that fewer than 25% exercise regularly

- Step 1 — Hypotheses: H■: $p = 0.25$, H■: $p < 0.25$ (left-tailed); p = proportion of adults who exercise regularly
- Step 2 — Conditions: $n \cdot p = 200 \times 0.25 = 50 > 10$ ✓; $n \cdot q = 200 \times 0.75 = 150 > 10$ ✓; sample is random ✓
- Step 3 — Test statistic: p■ = 44/200 = 0.22; $SE = \sqrt{(0.25 \times 0.75/200)} = \sqrt{(0.0009375)} \approx 0.03062$; $z = (0.22-0.25)/0.03062 \approx -0.98$
- Step 4 — Conclusion: p-value ≈ 0.1335 > 0.05 = α; fail to reject H■. There is not sufficient evidence to support the claim.

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10. Answer: Reject H■; sufficient evidence that the true proportion differs from 90%

- Step 1 — Hypotheses: H■: $p = 0.90$, H■: $p \neq 0.90$ (two-tailed); p = proportion of bulbs lasting more than 1,000 hours
- Step 2 — Conditions: $n \cdot p = 150 \times 0.90 = 135 > 10$ ✓; $n \cdot q = 150 \times 0.10 = 15 > 10$ ✓; random sample ✓
- Step 3 — Test statistic: p■ = 126/150 = 0.84; $SE = \sqrt{(0.90 \times 0.10/150)} = \sqrt{(0.0006)} \approx 0.02449$; $z = (0.84-0.90)/0.02449 \approx -2.45$
- Step 4 — Conclusion: p-value ≈ 0.0241 < 0.05 = α; reject H■. At the 5% significance level, there is sufficient evidence that the true proportion of bulbs lasting more than 1,000 hours differs from the manufacturer's claimed 90%.

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