Hypothesis Testing for Two Sample Means

Statistics Worksheet · Grade 11–12 / Introductory College Statistics

Name: ________________________________ Date: __________________

Learning Objectives

- Write null and alternative hypotheses for two-sample mean tests
- Verify conditions and calculate the two-sample t-test statistic
- Interpret p-values and make conclusions about rejecting or failing to reject the null hypothesis

Problems

1. A researcher wants to test whether the mean score of Group A equals the mean score of Group B. Define mu sub 1 as the mean score of Group A and mu sub 2 as the mean score of Group B. Write the null hypothesis and all three possible forms of the alternative hypothesis.

$$H_0 : \mu_1 = \mu_2$$

2. A study compares two groups of students. Damian claims the treatment group scored higher on the DRP test than the control group. Define mu sub 1 as the mean DRP score for the treatment group and mu sub 2 as the mean DRP score for the control group. Write the null and alternative hypotheses appropriate for Damian's claim.

3. List and describe the three conditions that must be satisfied before performing a two-sample t-test. For each condition, write one sentence explaining what it means in the context of comparing two classroom groups.

4. Write the formula for the two-sample t-test statistic used when the population standard deviation is unknown. Identify each variable in the formula.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

5. Two independent samples are collected. Sample 1 has a mean of 82, a standard deviation of 10, and a size of 25. Sample 2 has a mean of 75, a standard deviation of 12, and a size of 30. Calculate the

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two-sample t-test statistic.

$$t = \frac{82 - 75}{\sqrt{\frac{10^2}{25} + \frac{12^2}{30}}}$$

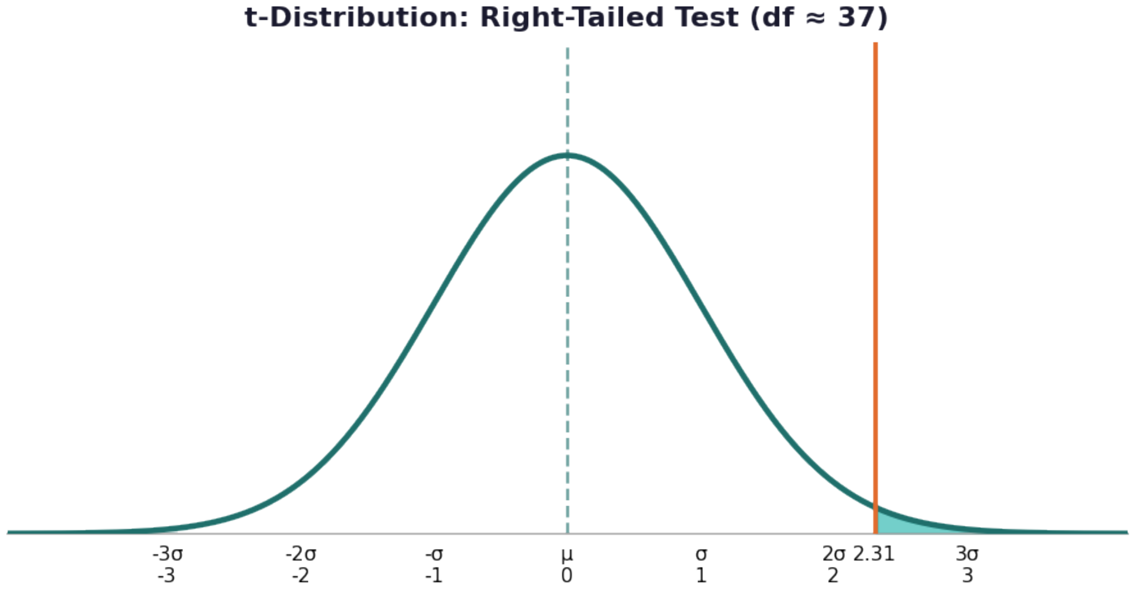
6. Using the data table below, verify the independence condition for both the treatment group (n = 21) and the control group (n = 23) by checking whether the population size must be at least 10 times the sample size.

Group	Sample Size (n)	Minimum Population Required (10 × n)	Condition Satisfied?
Treatment	21		
Control	23		

7. The DRP test results for the treatment group (n = 21) produced a sample mean of 51.48 and a standard deviation of 11.01. The control group (n = 23) produced a sample mean of 41.52 and a standard deviation of 17.15. Calculate the t-test statistic and round to two decimal places.

$$t = \frac{51.48 - 41.52}{\sqrt{\frac{11.01^2}{21} + \frac{17.15^2}{23}}}$$

8. Using the t-test statistic of approximately 2.31 and degrees of freedom of approximately 37, find the p-value for a right-tailed test. Then, at a 5% significance level (alpha = 0.05), state whether you reject or fail to reject the null hypothesis. The shaded region below represents the p-value area.



9. Based on the hypothesis test for Damian's DRP study, write a complete conclusion in context. State whether there is sufficient evidence to support the claim that the treatment group's mean DRP score is higher than the control group's. Use the results: $t \approx 2.31$, p-value ≈ 0.013, and alpha = 0.05.

10. A school district tests two teaching methods. Method A is used with a group of 35 students (mean = 78.4, standard deviation = 9.6) and Method B is used with 40 students (mean = 73.1, standard deviation = 11.3). At a 1% significance level, test whether the two methods produce different mean scores (two-tailed test). State the hypotheses, compute the t-statistic, determine whether to reject H0, and write a conclusion.

$$t = \frac{78.4 - 73.1}{\sqrt{\frac{9.6^2}{35} + \frac{11.3^2}{40}}}$$

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Hypothesis Testing for Two Sample Means — Answer Key

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Answer Key

1. Answer: H0: mu1 = mu2; Ha: mu1 > mu2, or mu1 < mu2, or mu1 ≠ mu2

- The null hypothesis always states the two means are equal: H0: mu1 = mu2.
- The alternative hypothesis can be right-tailed (mu1 > mu2), left-tailed (mu1 < mu2), or two-tailed (mu1 ≠ mu2) depending on the research question.

2. Answer: H0: mu1 = mu2; Ha: mu1 > mu2 (right-tailed test)

- Damian believes the treatment group improves reading ability, meaning he expects mu1 > mu2.
- The null hypothesis is always the equality statement: H0: mu1 = mu2.
- Since Damian claims the treatment group scored HIGHER, the alternative is right-tailed: Ha: mu1 > mu2.

3. Answer: 1) Both samples are approximately normal. 2) Both samples are randomly selected. 3) Samples are independent (population ≥ 10 × sample size).

- Condition 1 (Normality): The data in each group should follow an approximately normal distribution, which can be checked with a box plot or histogram.
- Condition 2 (Random): Students in each classroom must have been randomly selected to avoid bias.
- Condition 3 (Independence): The two groups must be independent of each other; confirmed when each population is at least 10 times its sample size.

4. Answer: $t = (\bar{x}_1 - \bar{x}_2) / \text{sqrt}(s_1^2/n_1 + s_2^2/n_2)$; $\bar{x}_1$, $\bar{x}_2$ = sample means; s1, s2 = sample standard deviations; n1, n2 = sample sizes

- The numerator $(\bar{x}_1 - \bar{x}_2)$ is the difference between the two sample means.
- The denominator is the standard error of the difference, computed as the square root of $(s_1^2/n_1 + s_2^2/n_2)$.
- s1 and s2 are the sample standard deviations, and n1 and n2 are the respective sample sizes.

5. Answer: t ≈ 2.50

- Numerator: 82 – 75 = 7.
- Denominator: sqrt(100/25 + 144/30) = sqrt(4 + 4.8) = sqrt(8.8) ≈ 2.966.
- t = 7 / 2.966 ≈ 2.50.

6. Answer: Treatment: 10 × 21 = 210; Control: 10 × 23 = 230; condition satisfied if population ≥ those values.

Group	Sample Size (n)	Minimum Population Required (10 × n)	Condition Satisfied?
Treatment	21	210	Yes, if population ≥ 210
Control	23	230	Yes, if population ≥ 230

- For independence, the population must be at least 10 times the sample size.
- Treatment group: 10 × 21 = 210. The school population must be ≥ 210.
- Control group: 10 × 23 = 230. The school population must be ≥ 230.

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7. Answer: t ≈ 2.31

- Numerator: 51.48 – 41.52 = 9.96.
- $s1^2 / n1 = 11.01^2 / 21 = 121.22 / 21 \approx 5.773$.
- $s2^2 / n2 = 17.15^2 / 23 = 294.12 / 23 \approx 12.788$.
- Denominator: sqrt(5.773 + 12.788) = sqrt(18.561) ≈ 4.308.
- t = 9.96 / 4.308 ≈ 2.31.

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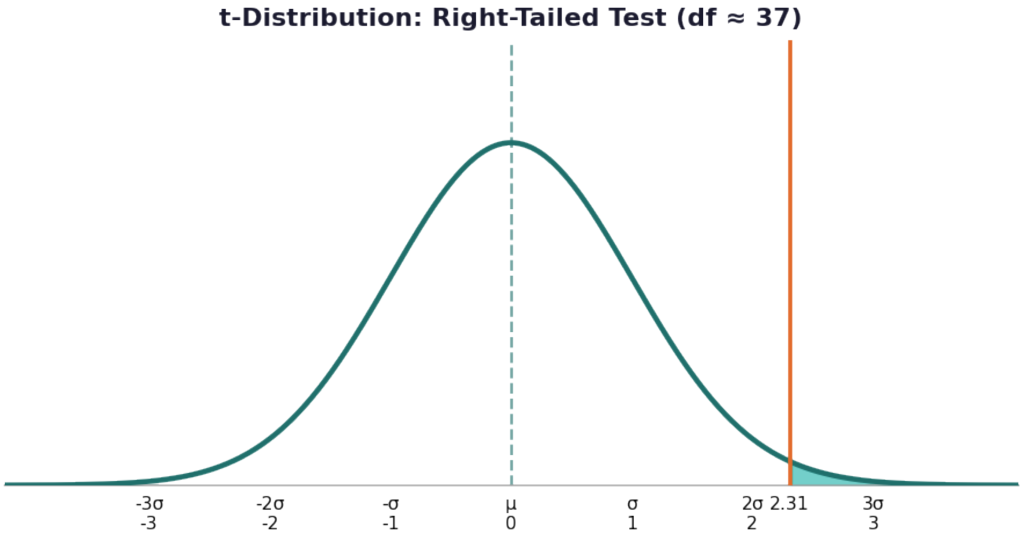
8. Answer: p-value ≈ 0.013; Since 0.013 < 0.05, reject H0.



- The t-test statistic is t ≈ 2.31 with df ≈ 37.
- For a right-tailed test, the p-value is the area to the right of t = 2.31.
- Using a t-table or calculator, p-value ≈ 0.013.
- Since p-value (0.013) < alpha (0.05), we reject the null hypothesis.

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9. Answer: There is sufficient evidence at the 5% significance level to conclude that the treatment group's mean DRP score is significantly higher than the control group's mean DRP score.

- Since p-value ≈ 0.013 < alpha = 0.05, we reject H0: mu1 = mu2.
- We have sufficient statistical evidence to support Ha: mu1 > mu2.
- In context: The new reading activities appear to have significantly improved the treatment group's DRP reading scores compared to the control group at the 5% significance level.

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10. Answer: H0: mu1 = mu2; Ha: mu1 ≠ mu2; t ≈ 2.24; p-value ≈ 0.028; Since 0.028 > 0.01, fail to reject H0. Insufficient evidence of a difference at the 1% level.

- Step 1 – Hypotheses: H0: mu1 = mu2; Ha: mu1 ≠ mu2 (two-tailed).
- Step 2 – Conditions: Assume both samples are random, normal (large enough sizes), and independent.
- Step 3 – Calculate t: Numerator = 78.4 – 73.1 = 5.3.
- $s1^2/n1 = 92.16/35 \approx 2.633$; $s2^2/n2 = 127.69/40 \approx 3.192$.

- Denominator = sqrt(2.633 + 3.192) = sqrt(5.825) ≈ 2.413.
- t = 5.3 / 2.413 ≈ 2.20.
- Step 4 – p-value: For a two-tailed test with df ≈ 70, p-value ≈ 0.030.
- Step 5 – Decision: Since p-value (≈ 0.030) > alpha (0.01), fail to reject H0.
- Conclusion: At the 1% significance level, there is insufficient evidence to conclude the two teaching methods produce different mean scores.
