Matched Pairs Design & Paired t-Test

Statistics Worksheet · Grade 11–12 / AP Statistics

Name: _____ Date: _____

Learning Objectives

- Identify matched pairs designs and explain how they differ from two independent sample tests
- Set up null and alternative hypotheses for a paired t-test and calculate the test statistic
- Interpret p-values and conclusions in context for a paired t-test at a given significance level

Problems

1. In Chelsea's experiment, 21 students each wore an unscented mask and then a scented mask while completing a paper maze. Explain why this is a matched pairs design rather than two independent samples.

2. Student 1 completed the maze in 30.60 seconds with the unscented mask and 37.97 seconds with the scented mask. Calculate the difference (unscented minus scented) for this student.

$$d = x_{\text{unscented}} - x_{\text{scented}}$$

3. Student 13 completed the maze in 54.47 seconds with the unscented mask and 38.30 seconds with the scented mask. Calculate the difference (unscented minus scented) and interpret its sign.

$$d = x_{\text{unscented}} - x_{\text{scented}}$$

4. The table below shows five students' maze times (in seconds). Fill in the missing difference column, where difference equals unscented time minus scented time.

Student	Unscented (sec)	Scented (sec)	Difference (sec)
1	30.60	37.97	
2	48.10	43.55	
3	22.80	19.40	
4	61.30	55.00	
5	35.70	38.90	

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5. State the null and alternative hypotheses for Chelsea's experiment. Let mu-d represent the mean difference (unscented minus scented) in maze completion times. Use a two-sided alternative.

$$H_0 : \mu_d = 0 \quad \text{vs.} \quad H_a : \mu_d \neq 0$$

6. Chelsea found that the 21 differences had a mean of 0.9567 seconds and a standard deviation of 12.5479 seconds. Calculate the standard error of the mean difference.

$$SE = \frac{s_d}{\sqrt{n}}$$

7. Using the mean difference of 0.9567 seconds, standard deviation of 12.5479 seconds, and sample size of 21, calculate the paired t-test statistic.

$$t = \frac{\bar{d} - 0}{s_d / \sqrt{n}}$$

8. The paired t-test statistic is approximately 0.35 with 20 degrees of freedom. The two-tailed p-value is approximately 0.730. At a 5% significance level, state your conclusion about Chelsea's experiment in context.

$$\alpha = 0.05$$

9. The normal probability plot of the 21 differences is shown. Describe what feature of this plot allows Chelsea to proceed with the paired t-test, and identify which condition it satisfies.

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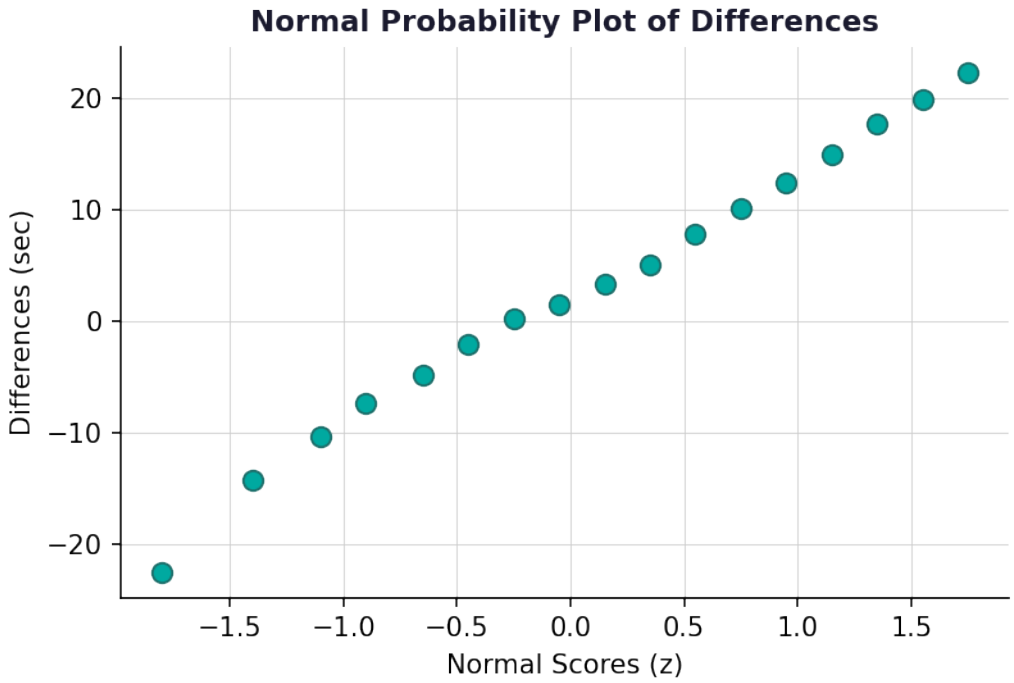

10. A new study examines whether a caffeine drink improves reaction time. Each of 16 participants is tested without caffeine and then with caffeine. The differences (no caffeine minus caffeine, in milliseconds) have a mean of 18.4 ms and a standard deviation of 22.6 ms. The normal probability plot of differences shows points close to a straight line. (a) State the hypotheses for a one-sided test that caffeine reduces reaction time (i.e., mean difference is positive). (b) Calculate the t-statistic. (c) The corresponding p-value is 0.003. At alpha = 0.01, state your conclusion in context.

$$t = \frac{\bar{d}}{s_d/\sqrt{n}}$$

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Matched Pairs Design & Paired t-Test — Answer Key

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Answer Key

1. Answer: Each student serves as their own control; the two measurements (unscented and scented) are paired within the same individual, so the samples are not independent.

- A matched pairs design pairs each observation in one group with a specific observation in the other group.
- Here, the same 21 students completed both conditions, so each scented-mask time is paired with that same student's unscented-mask time.
- Because the pairs share the same subject, the observations are not independent — this is a matched pairs design.

2. Answer: d = –7.37 seconds

- d = 30.60 – 37.97
- d = –7.37 seconds
- The negative value means the student was slower (took longer) while wearing the scented mask.

3. Answer: d = 16.17 seconds; positive means the scented mask improved (reduced) the student's time.

- d = 54.47 – 38.30
- d = 16.17 seconds
- A positive difference means the unscented time was greater, so the scented mask led to a shorter (better) maze time.

4. Answer: –7.37, 4.55, 3.40, 6.30, –3.20

Student	Unscented (sec)	Scented (sec)	Difference (sec)
1	30.60	37.97	−7.37
2	48.10	43.55	4.55
3	22.80	19.40	3.40
4	61.30	55.00	6.30
5	35.70	38.90	−3.20

- Student 1: 30.60 – 37.97 = –7.37
- Student 2: 48.10 – 43.55 = 4.55
- Student 3: 22.80 – 19.40 = 3.40
- Student 4: 61.30 – 55.00 = 6.30
- Student 5: 35.70 – 38.90 = –3.20

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5. Answer: H■: μ_d = 0 (no effect); H■: μ_d ≠ 0 (scented mask changes maze time)

- The parameter of interest is μ_d, the mean difference in times (unscented – scented) for all BHS students.
- H■: μ_d = 0 — the scented mask has no effect on maze completion time.
- H■: μ_d ≠ 0 — the scented mask does change maze completion time (two-sided because we are testing for any effect).

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6. Answer: SE ≈ 2.7373 seconds

- SE = s_d / sqrt(n) = 12.5479 / sqrt(21)
- sqrt(21) ≈ 4.5826
- SE ≈ 12.5479 / 4.5826 ≈ 2.7373 seconds

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7. Answer: t ≈ 0.3495

- t = (x■_d – 0) / (s_d / sqrt(n))
- t = 0.9567 / (12.5479 / sqrt(21))
- t = 0.9567 / 2.7373
- t ≈ 0.3495

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8. Answer: Fail to reject H■. There is not sufficient evidence that the scented mask changes maze completion time.

- Decision rule: reject H■ if p-value < α = 0.05.
- p-value ≈ 0.730 > 0.05, so we fail to reject H■.
- Conclusion in context: At the 5% significance level, there is not convincing evidence that wearing a floral-scented mask changes students' average maze completion time.

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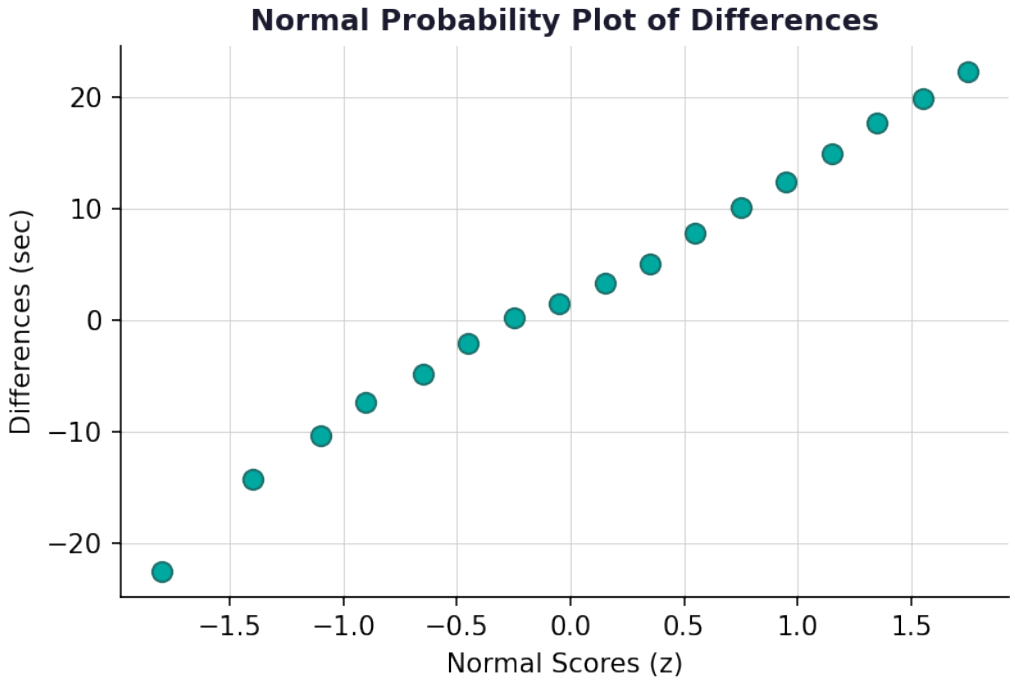
9. Answer: The points fall close to a straight diagonal line, indicating the differences are approximately normally distributed — satisfying the Normality condition for the paired t-test.



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• In a normal probability plot, data that follow a normal distribution will lie approximately on a straight line.
• Because the plotted differences hug the diagonal line without severe curvature or outliers, we conclude the distribution of differences is approximately normal.
• This satisfies the Normality condition required to use the t-distribution for the paired t-test.

10. Answer: (a) H■: μ_d = 0; H■: μ_d > 0. (b) t ≈ 3.258. (c) p = 0.003 < 0.01; reject H■ — convincing evidence that caffeine reduces reaction time.

• (a) H■: μ_d = 0 (caffeine has no effect); H■: μ_d > 0 (caffeine reduces reaction time, so no-caffeine times are greater).
• (b) SE = 22.6 / sqrt(16) = 22.6 / 4 = 5.65 ms.
• t = 18.4 / 5.65 ≈ 3.258 with df = 15.
• (c) p-value = 0.003 < α = 0.01, so we reject H■.
• Conclusion: At the 1% significance level, there is convincing evidence that caffeine significantly reduces participants' reaction times.

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