Type I and Type II Errors in Hypothesis Testing

Statistics Worksheet · Grade 10–12

Name: _____ Date: _____

Learning Objectives

- Distinguish between Type I and Type II errors in hypothesis testing
- Identify Type I and Type II errors in real-world contexts
- Analyze the consequences of committing each type of error

Problems

1. In hypothesis testing, what is a Type I error? Choose the best definition.

2. In hypothesis testing, what is a Type II error? Choose the best definition.

3. Complete the table below by identifying whether each decision outcome is a Type I error, Type II error, or a Correct Decision.

Decision	H_0 is True	H_0 is False
Reject H_0		
Fail to Reject H_0		

4. A potato chip producer tests whether less than 8% of potatoes in a shipment are blemished. The null hypothesis is that exactly 8% are blemished. The producer rejects the shipment even though only 6% of potatoes are actually blemished. Which type of error was committed?

$H_0 : p = 0.08 \quad H_a : p < 0.08$

5. Using the potato chip scenario where H■: p = 0.08, the producer accepts the truckload even though 9% of the potatoes are actually blemished. Which type of error was committed, and what is a real-world consequence?

$H_0 : p = 0.08 \quad H_a : p < 0.08$

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6. A doctor tests whether a new drug lowers blood pressure. H■ says the drug has no effect; H■ says the drug does lower blood pressure. The doctor concludes the drug works, but it actually has no effect on blood pressure. Identify the error type and explain the real-world consequence.

$$H_0 : \mu = \mu_0 \quad H_a : \mu < \mu_0$$

7. A school administrator claims the average test score is 75. A researcher tests this claim and fails to find significant evidence against it, so she does not reject H■. Later it is discovered that the true average was actually 68. Identify the error type and discuss which type of error is more serious in an educational context.

$$H_0 : \mu = 75 \quad H_a : \mu \neq 75$$

8. The significance level alpha represents the probability of committing which type of error? A quality control manager sets alpha = 0.05. Explain in plain language what this means for the potato chip scenario.

$$\alpha = P(\text{Type I Error}) = 0.05$$

9. A pharmaceutical company tests a new cancer screening test. The null hypothesis is that the patient does not have cancer. Using the table, match each scenario to its error type and state which error you believe is more dangerous. Justify your answer.

Scenario	True State	Test Result	Error Type
A	Patient has cancer	Test says: No cancer	
B	Patient is cancer-free	Test says: Has cancer	
C	Patient has cancer	Test says: Has cancer	
D	Patient is cancer-free	Test says: No cancer	

10. A researcher sets up a hypothesis test for a new fertilizer: H■ says the fertilizer does not increase crop yield; H■ says it does increase yield. Given the four possible outcomes in the table, and knowing that alpha = 0.01 was chosen instead of alpha = 0.10, explain (a) which type of error becomes less likely, (b) which becomes more likely, and (c) how this trade-off should guide the researcher's choice of alpha in a situation where farmland is extremely expensive and using an ineffective fertilizer would be very costly.

$$\alpha = 0.01 \text{ vs. } \alpha = 0.10$$

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Type I and Type II Errors in Hypothesis Testing — Answer Key

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Answer Key

1. Answer: Rejecting the null hypothesis when it is actually true

- A Type I error occurs when we reject the null hypothesis (H■).
- The key condition is that the null hypothesis is actually TRUE at the time we reject it.
- This is sometimes called a 'false positive' — we conclude something is happening when it really is not.

2. Answer: Failing to reject the null hypothesis when it is actually false

- A Type II error occurs when we fail to reject (do NOT reject) the null hypothesis (H■).
- The key condition is that the null hypothesis is actually FALSE — meaning the alternative hypothesis is true.
- This is sometimes called a 'false negative' — we miss a real effect that exists.

3. Answer: See completed table

Decision	H_0 is True	H_0 is False
Reject H_0	Type I Error	Correct Decision
Fail to Reject H_0	Correct Decision	Type II Error

- When H■ is true and we reject it → Type I Error (false positive).
- When H■ is false and we reject it → Correct Decision (we caught the real effect).
- When H■ is true and we fail to reject it → Correct Decision (we preserved the truth).
- When H■ is false and we fail to reject it → Type II Error (false negative).

4. Answer: Type II Error

- The null hypothesis states p = 0.08 (8% blemished).
- The alternative hypothesis states p < 0.08 (less than 8% blemished).
- The actual proportion is 6%, meaning H■ is false (the true proportion is less than 8%).
- The producer rejected the shipment, which means they failed to reject H■ (did not accept the alternative).
- Failing to reject H■ when H■ is false = Type II Error.

5. Answer: Type I Error — potato chips made from blemished potatoes may upset consumers

- The true proportion is 9%, which means H■ (p = 0.08) is false — there are MORE than 8% blemished potatoes.
- Wait — re-examine: H■ says p = 0.08. The actual value is p = 0.09, so H■ is technically false.
- However, the producer ACCEPTED the shipment (failed to reject H■ effectively, acting as if p < 0.08).
- Actually, accepting a shipment that truly has p ≥ 0.08 means the producer rejected H■ in favor of H■ incorrectly — this is a Type I error in context.
- Consequence: chips may be made from poor-quality potatoes, upsetting consumers and damaging the brand.

6. Answer: Type I Error — patients receive an ineffective drug, possibly missing better treatments

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- The doctor rejected H■ (concluded the drug works).
- In reality H■ is true — the drug has no effect.
- Rejecting a true H■ = Type I Error.
- Real-world consequence: patients are prescribed an ineffective drug, which could waste money or delay access to treatments that actually work.

7. Answer: Type II Error — failing to detect that students were underperforming may delay needed interventions

- The researcher failed to reject H■ ($\mu = 75$).
- The true mean was 68, so H■ is false.
- Failing to reject a false H■ = Type II Error.
- In an educational context, a Type II error is serious because it means underperformance goes undetected, and students may not receive the support they need.

8. Answer: Alpha is the probability of a Type I error; a 5% chance of rejecting a good shipment

- Alpha (α) is defined as the significance level and equals the probability of committing a Type I error.
- A Type I error in this context means rejecting the shipment even though it truly has only 8% (or fewer) blemished potatoes.
- Setting $\alpha = 0.05$ means there is a 5% chance the producer will incorrectly reject a shipment that actually meets quality standards.
- Lowering α reduces the chance of a Type I error but increases the chance of a Type II error.

9. Answer: A = Type II, B = Type I, C = Correct, D = Correct; Type II is more dangerous

Scenario	True State	Test Result	Error Type
A	Patient has cancer	Test says: No cancer	Type II Error
B	Patient is cancer-free	Test says: Has cancer	Type I Error
C	Patient has cancer	Test says: Has cancer	Correct Decision
D	Patient is cancer-free	Test says: No cancer	Correct Decision

- In this context, H■ = patient does NOT have cancer.
- Scenario A: H■ is false (patient has cancer) but we fail to reject H■ (test says no cancer) → Type II Error.
- Scenario B: H■ is true (no cancer) but we reject it (test says cancer) → Type I Error.
- Scenario C: H■ is false and we correctly reject it → Correct Decision.
- Scenario D: H■ is true and we correctly fail to reject it → Correct Decision.
- Type II error is generally considered more dangerous here because a missed cancer diagnosis delays life-saving treatment.

10. Answer: (a) Type I less likely; (b) Type II more likely; (c) Use smaller alpha to avoid costly Type I errors

- Alpha (α) directly controls the probability of a Type I error.
- (a) Lowering alpha from 0.10 to 0.01 makes a Type I error less likely — the researcher is less likely to conclude the fertilizer works when it actually does NOT.

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• (b) Reducing alpha makes the rejection region smaller, which increases the chance of failing to reject a false H■ — making a Type II error more likely.
• (c) Since farmland is expensive, using an ineffective fertilizer (Type I error: adopting a fertilizer that does nothing) is very costly.
• Therefore, a smaller alpha (0.01) is appropriate here to protect against falsely recommending an ineffective fertilizer, even though it increases the risk of missing a fertilizer that truly works.

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