

# Chi-Square Test for Independence

Statistics Worksheet · Grade 11–College

Name: \_\_\_\_\_

Date: \_\_\_\_\_

## Learning Objectives

- Identify when a chi-square test for independence is appropriate and state correct null and alternative hypotheses
- Compute expected counts, the chi-square test statistic, and degrees of freedom from a two-way contingency table
- Verify conditions for the chi-square test and draw conclusions based on p-value comparisons

## Problems

1. A researcher surveys a random sample of adults and records both their political party (Democrat, Republican, Independent) and their opinion on a new tax policy (Favor, Oppose). Which type of chi-square test should be used, and why?

2. A survey on handgun ban opinion cross-classified 1,201 adults by education level and response (Yes or No). State the null and alternative hypotheses for the chi-square test for independence.

3. A two-way table has 3 rows and 4 columns. Calculate the degrees of freedom for a chi-square test of independence.

$$df = (r - 1)(c - 1)$$

4. The observed two-way table below shows survey counts for opinion on a handgun ban by education level. Complete the row totals, column totals, and grand total.

Education Level	Yes	No	Row Total

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5. Using the completed table from the previous problem, verify the three conditions required before performing a chi-square test for independence: randomness, the 10% condition, and large counts (expected counts  $\geq 5$ ).

6. Calculate the expected count for the cell corresponding to 'Less than HS' education and 'Yes' response, using the totals: row total = 95, column total = 446, and grand total = 1201.

$$E = \frac{(\text{Row Total}) \times (\text{Column Total})}{\text{Grand Total}}$$

7. The observed count for a cell is 42 and its expected count is 35.28. Calculate the chi-square contribution for this single cell.

$$\chi^2_{\text{cell}} = \frac{(O - E)^2}{E}$$

8. Using the observed and expected counts table below, calculate the total chi-square test statistic by summing all cell contributions.

Cell	O	E	$\frac{(O-E)^2}{E}$

9. The chi-square test statistic for the handgun ban survey is approximately 26.78 with 4 degrees of freedom. The p-value is less than 0.0001. Using a significance level of 5%, state the conclusion of the hypothesis test in context.

$$\alpha = 0.05, \quad \chi^2 \approx 26.78, \quad df = 4, \quad p < 0.0001$$

10. A different study surveys two separate groups — one group of gun owners and one group of non-gun-owners — and records whether each person supports or opposes stricter background checks. A

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student claims this should also use a chi-square test for independence. Is the student correct? Identify the correct test and explain the key distinction.

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# Chi-Square Test for Independence — Answer Key

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## Answer Key

**1. Answer: Chi-square test for independence; one random sample is classified according to two categorical variables.**

- Identify the sampling design: one single random sample of adults.
- Each individual is classified by two variables: political party and tax opinion.
- When a single sample is cross-classified by two categorical variables, the appropriate test is the chi-square test for independence.

**2. Answer: H0: There is no association between education level and opinion about a handgun ban in the adult population. Ha: There is an association between education level and opinion about a handgun ban in the adult population.**

- The null hypothesis always states independence (no association) between the two categorical variables.
- The alternative hypothesis states that an association exists.
- H0: Education level and handgun ban opinion are independent in the adult population.
- Ha: Education level and handgun ban opinion are associated in the adult population.

**3. Answer: df = 6**

- The formula for degrees of freedom is  $df = (\text{number of rows} - 1)(\text{number of columns} - 1)$ .
- $df = (3 - 1)(4 - 1) = (2)(3) = 6$ .

**4. Answer: See completed table**

Education Level	Yes	No	Row Total
Less than HS	42	53	95
HS Graduate	188	230	418
Some College	90	159	249
College Grad	70	181	251

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Education Level	Yes	No	Row Total
Postgraduate	56	132	188
Column Total	446	755	1201

**5. Answer: All three conditions are satisfied: random sample stated, n = 1201 is less than 10% of all adults, and all expected counts exceed 5.**

- Randomness: The problem states a random sample of adults was selected — satisfied.
- 10% condition: The total population of adults is far greater than  $10 \times 1201 = 12,010$  — satisfied.
- Large counts: Each expected count = (row total  $\times$  column total) / grand total must be  $\geq 5$ . The smallest row total is 95; its smallest expected count =  $(95 \times 446) / 1201 \approx 35.3 \geq 5$  — satisfied.

**6. Answer: E  $\approx$  35.30**

- Apply the expected count formula:  $E = (\text{row total} \times \text{column total}) / \text{grand total}$ .
- $E = (95 \times 446) / 1201 = 42,370 / 1201 \approx 35.28$ .
- Round to two decimal places:  $E \approx 35.28$  (approximately 35.30).

**7. Answer:  $\approx$  1.28**

- $O = 42, E = 35.28$ .
- $(O - E)^2 = (42 - 35.28)^2 = (6.72)^2 = 45.1584$ .
- Divide by E:  $45.1584 / 35.28 \approx 1.28$ .

**8. Answer:  $\chi^2 \approx$  26.78**

Cell	O	E	$(O-E)^2 / E$
Less than HS / Yes	42	35.28	1.28
Less than HS / No	53	59.72	0.76
HS Grad / Yes	188	155.16	6.95

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Cell	O	E	$\frac{(O-E)^2}{E}$
HS Grad / No	230	262.84	4.10
Some College / Yes	90	92.45	0.06
Some College / No	159	156.55	0.04
College Grad / Yes	70	93.21	5.79
College Grad / No	181	157.79	3.42
Postgrad / Yes	56	69.87	2.75
Postgrad / No	132	118.13	1.63
Total			26.78

**9. Answer: Reject H0. There is convincing evidence of an association between education level and opinion about a handgun ban in the adult population.**

- Compare p-value to significance level:  $p < 0.0001 < 0.05 = \alpha$ .
- Since  $p < \alpha$ , reject the null hypothesis.
- Conclusion: The data provide convincing evidence that education level and opinion about a handgun ban are associated in the adult population.

**10. Answer: No. This is a chi-square test for homogeneity because two separate samples (gun owners and non-gun-owners) were drawn independently, not one single sample classified by two variables.**

- Test for independence: one random sample is collected, and each individual is classified according to two categorical variables.
- Test for homogeneity: two or more separate random samples are drawn from distinct populations and compared on one categorical variable.

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- Here, two distinct groups (gun owners and non-gun-owners) were sampled separately — this matches the test for homogeneity.
  - The student is incorrect; the appropriate test is the chi-square test for homogeneity.
  - Note: the calculation procedure is identical for both tests — only the study design and hypotheses differ.
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