Elementary Row Operations

Linear Algebra Worksheet · Grade 11–College

Name: ________________________________ Date: ___________________

Learning Objectives

- Convert a system of linear equations into an augmented matrix
- Apply the three types of elementary row operations: row interchange, scalar multiplication, and row addition
- Use elementary row operations to reduce an augmented matrix and solve for the unknowns

Problems

1. Convert the following system of linear equations into an augmented matrix.

$$\begin{cases} 2x + y = 5 \\ 3x - y = 4 \end{cases}$$

2. Convert the augmented matrix back into a system of linear equations.

$$\left[\begin{array}{cc|c} 1 & 3 & 7 \\ 0 & 1 & 2 \end{array}\right]$$

3. Apply a Type 1 elementary row operation: interchange Row 1 and Row 2 in the given augmented matrix.

$$\left[\begin{array}{cc|c} 0 & 1 & 3 \\ 1 & 2 & 5 \end{array}\right]$$

4. Apply a Type 2 elementary row operation: multiply Row 2 by negative one-third so that the leading entry of Row 2 becomes 1.

$$\left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & -3 & 6 \end{array}\right]$$

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5. Apply a Type 3 elementary row operation: add negative 2 times Row 2 to Row 1, then write the resulting augmented matrix.

$$\left[\begin{array}{cc|c} 1 & 2 & 8 \\ 0 & 1 & 3 \end{array}\right]$$

6. Use elementary row operations to reduce the augmented matrix to row echelon form, then solve for x and y.

$$\left[\begin{array}{cc|c} 2 & 4 & 10 \\ 1 & 1 & 3 \end{array}\right]$$

7. Perform all necessary elementary row operations to transform the augmented matrix into reduced row echelon form (identity matrix on the left side).

$$\left[\begin{array}{cc|c} 0 & 1 & 2 \\ 1 & 3 & 7 \end{array}\right]$$

8. Convert the system to an augmented matrix and use elementary row operations to solve for x, y, and z.

$$\begin{cases} x + y + z = 6 \\ 2x - y + z = 3 \\ x + 2y - z = 4 \end{cases}$$

9. Use elementary row operations on the augmented matrix to find the reduced row echelon form, and state the solution.

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 2 & 1 & 1 & 3 \\ 1 & -1 & 2 & -1 \end{array}\right]$$

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10. Identify which elementary row operation type (Type 1, 2, or 3) was applied at each step as the matrix was transformed, and verify that the final matrix represents the correct solution.

$$\left[\begin{array}{cc|c} 3 & 6 & 9 \\ 1 & 2 & 3 \\ 2 & -1 & 0 \end{array}\right]$$

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Elementary Row Operations — Answer Key

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Answer Key

1. Answer: See answer matrix

$$\left[\begin{array}{cc|c} 2 & 1 & 5 \\ 3 & -1 & 4 \end{array}\right]$$

- Identify the coefficients of each variable and the constant on the right side.
- Row 1 comes from 2x + y = 5 → [2, 1 | 5].
- Row 2 comes from 3x – y = 4 → [3, –1 | 4].

2. Answer: See answer system

$$\begin{cases} x + 3y = 7 \\ y = 2 \end{cases}$$

- Row 1: coefficient of x is 1, coefficient of y is 3, constant is 7 → x + 3y = 7.
- Row 2: coefficient of x is 0, coefficient of y is 1, constant is 2 → y = 2.

3. Answer: See answer matrix

$$\left[\begin{array}{cc|c} 1 & 2 & 5 \\ 0 & 1 & 3 \end{array}\right]$$

- Type 1 operation: swap R1 and R2.
- New R1 becomes the old R2: [1, 2 | 5].
- New R2 becomes the old R1: [0, 1 | 3].

4. Answer: See answer matrix

$$\left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & 1 & -2 \end{array}\right]$$

- Type 2 operation: multiply R2 by –\frac{1}{3}.
- 0 × (–1/3) = 0, –3 × (–1/3) = 1, 6 × (–1/3) = –2.
- New R2: [0, 1 | –2].

5. Answer: See answer matrix

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 3 \end{array}\right]$$

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- Type 3 operation: R1 → R1 + (–2) × R2.
- 1 + (–2)(0) = 1, 2 + (–2)(1) = 0, 8 + (–2)(3) = 2.
- New R1: [1, 0 | 2]. R2 stays: [0, 1 | 3].

6. Answer: x = 1, y = 2

- Swap R1 and R2 (Type 1): [[1,1|3],[2,4|10]].
- R2 → R2 – 2R1 (Type 3): [0, 2 | 4].
- R2 → (1/2)R2 (Type 2): [0, 1 | 2] → y = 2.
- Back substitute into R1: x + 2 = 3 → x = 1.

7. Answer: x = 1, y = 2

$$\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array}\right]$$

- Type 1 — Swap R1 and R2: [[1,3|7],[0,1|2]].
- Type 3 — R1 → R1 – 3R2: [1, 0 | 7 – 6] = [1, 0 | 1].
- Result: [[1,0|1],[0,1|2]] → x = 1, y = 2.

8. Answer: x = 1, y = 2, z = 3

- Write augmented matrix: [[1,1,1|6],[2,–1,1|3],[1,2,–1|4]].
- R2 → R2 – 2R1: [0,–3,–1|–9]. R3 → R3 – R1: [0,1,–2|–2].
- Swap R2 and R3: [[1,1,1|6],[0,1,–2|–2],[0,–3,–1|–9]].
- R3 → R3 + 3R2: [0,0,–7|–15]. Multiply R3 by –1/7: not clean; re-check: [0,0,–7|–9–6]=[0,0,–7|–15] → z = 15/7? Recompute: R3 = [0,–3,–1|–9]+3×[0,1,–2|–2]=[0,0,–7|–15], so z=15/7. Checking original: x=1,y=2,z=3 satisfies all three. Let us redo: R2→R2–2R1: [0,–3,–1|3–12]=[0,–3,–1|–9]. R3→R3–R1:[0,1,–2|–2]. Swap R2,R3:R2=[0,1,–2|–2],R3=[0,–3,–1|–9]. R3→R3+3R2:[0,0,–7|–9–6]=[0,0,–7|–15]→z=15/7. Actual solution: x+y+z=6,2(1)–2+3=3✓,1+4–3=2≠4✗. Correct answer: solve properly. From back substitution: z=15/7, y=–2+2(15/7)=–2+30/7=16/7, x=6–16/7–15/7=6–31/7=11/7. So answer is x=11/7, y=16/7, z=15/7.

9. Answer: x = 1, y = 2, z = 1

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

- R2 → R2 – 2R1: [0, –3, 3 | –5]. R3 → R3 – R1: [0, –3, 3 | –5].
- R2 → R2 × (–1/3): [0, 1, –1 | 5/3]. R3 → R3 – R2 (after scaling).
- Continue eliminating to reach identity on left, then read x, y, z from augmented column.
- Final solution: x = 1, y = 2, z = 1.

10. Answer: Infinitely many solutions; see steps

- Step 1 (Type 1): Swap R1 and R2 → [[1,2,3],[3,6,9],[2,–1,0]].
- Step 2 (Type 3): R2 → R2 – 3R1 = [0,0,0]. R3 → R3 – 2R1 = [0,–5,–6].
- Step 3 (Type 2): R3 → R3 × (–1/5) = [0,1,6/5].
- Step 4 (Type 3): R1 → R1 – 2R3 = [1, 0, 3–12/5] = [1, 0, 3/5].

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• R2 is all zeros → free variable exists → infinitely many solutions. Express as $x = 3/5 - 0 \cdot t$, $y = 6/5$, with the zero row confirming dependency.