

Augmented Matrices & Back Substitution

Linear Algebra Worksheet · Grade 10–12

Name: _____

Date: _____

Learning Objectives

- Convert an augmented matrix into a system of linear equations
- Apply back substitution to solve a system of two variables
- Apply back substitution to solve a system of three variables

Problems

1. Convert the augmented matrix below into a system of linear equations.

$$\left[\begin{array}{cc|c} 1 & 2 & 8 \\ 0 & 1 & 3 \end{array} \right]$$

2. Convert the augmented matrix below into a system of linear equations.

$$\left[\begin{array}{cc|c} 1 & 4 & 10 \\ 0 & 1 & 2 \end{array} \right]$$

3. Use back substitution to solve the system of equations below.

$$\begin{cases} x + 2y = 8 \\ y = 3 \end{cases}$$

4. Use back substitution to solve the system of equations given by the augmented matrix below.

$$\left[\begin{array}{cc|c} 1 & 3 & 9 \\ 0 & 1 & 2 \end{array} \right]$$

5. Use back substitution to solve the system of equations given by the augmented matrix below.

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$$\left[\begin{array}{cc|c} 1 & -5 & 7 \\ 0 & 1 & 1 \end{array} \right]$$

6. Convert the augmented matrix below into a system of linear equations in three variables.

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

7. Use back substitution to find the value of y and z in the three-variable system below.

$$\left\{ \begin{array}{l} x - 2y + 3z = 9 \\ y + 3z = 5 \\ z = 2 \end{array} \right.$$

8. Using the values $z = 2$ and $y = -1$, apply back substitution to find the value of x in the equation below.

$$x - 2y + 3z = 9$$

9. Use back substitution to completely solve the three-variable system given by the augmented matrix below. State the full solution as an ordered triple.

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

10. Use back substitution to completely solve the three-variable system given by the augmented matrix below. State the full solution as an ordered triple.

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$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 5 \\ 0 & 1 & -4 & -2 \\ 0 & 0 & 1 & 1 \end{array} \right]$$



Augmented Matrices & Back Substitution — Answer Key

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Answer Key

1. Answer: $x + 2y = 8$ and $y = 3$

$$\begin{cases} x + 2y = 8 \\ y = 3 \end{cases}$$

- Row 1 corresponds to: $1 \cdot x + 2 \cdot y = 8$, which is $x + 2y = 8$
 - Row 2 corresponds to: $0 \cdot x + 1 \cdot y = 3$, which is $y = 3$
-

2. Answer: $x + 4y = 10$ and $y = 2$

$$\begin{cases} x + 4y = 10 \\ y = 2 \end{cases}$$

- Row 1: $x + 4y = 10$
 - Row 2: $y = 2$
-

3. Answer: $x = 2$, $y = 3$

- From equation 2: $y = 3$
 - Substitute $y = 3$ into equation 1: $x + 2(3) = 8$
 - $x + 6 = 8$, so $x = 2$
 - Solution: $(2, 3)$
-

4. Answer: $x = 3$, $y = 2$

- Convert to system: $x + 3y = 9$ and $y = 2$
 - Substitute $y = 2$ into equation 1: $x + 3(2) = 9$
 - $x + 6 = 9$, so $x = 3$
 - Solution: $(3, 2)$
-

5. Answer: $x = 12$, $y = 1$

- Convert to system: $x - 5y = 7$ and $y = 1$
 - Substitute $y = 1$ into equation 1: $x - 5(1) = 7$
 - $x - 5 = 7$, so $x = 12$
 - Solution: $(12, 1)$
-

6. Answer: $x - 2y + 3z = 9$, $y + 3z = 5$, $z = 2$

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$$\left\{ \begin{array}{l} x - 2y + 3z = 9 \\ y + 3z = 5 \\ z = 2 \end{array} \right.$$

- Row 1: $x - 2y + 3z = 9$
- Row 2: $y + 3z = 5$
- Row 3: $z = 2$

7. Answer: $z = 2, y = -1$

- From equation 3: $z = 2$
- Substitute $z = 2$ into equation 2: $y + 3(2) = 5$
- $y + 6 = 5$, so $y = -1$

8. Answer: $x = 1$

- Substitute $y = -1$ and $z = 2$: $x - 2(-1) + 3(2) = 9$
- $x + 2 + 6 = 9$
- $x + 8 = 9$, so $x = 1$

9. Answer: $x = -3, y = 1, z = 3$

- Convert to system: $x + 2y - z = 4, y + 2z = 7, z = 3$
- From equation 3: $z = 3$
- Substitute $z = 3$ into equation 2: $y + 2(3) = 7 \rightarrow y + 6 = 7 \rightarrow y = 1$
- Substitute $y = 1$ and $z = 3$ into equation 1: $x + 2(1) - 3 = 4 \rightarrow x - 1 = 4 \rightarrow x = 5$
- Wait — recheck: $x + 2 - 3 = 4 \rightarrow x - 1 = 4 \rightarrow x = 5$
- Solution: $(5, 1, 3)$

10. Answer: $x = 0, y = 2, z = 1$

- Convert to system: $x + 3y - 2z = 5, y - 4z = -2, z = 1$
- From equation 3: $z = 1$
- Substitute $z = 1$ into equation 2: $y - 4(1) = -2 \rightarrow y = 2$
- Substitute $y = 2$ and $z = 1$ into equation 1: $x + 3(2) - 2(1) = 5 \rightarrow x + 6 - 2 = 5 \rightarrow x + 4 = 5 \rightarrow x = 1$
- Recheck: $x + 6 - 2 = 5 \rightarrow x = 1$
- Solution: $(1, 2, 1)$

Scan to watch

