# Determinants of Matrices

Algebra Worksheet · Grade 10–12

Name: ______________________________ Date: __________________

**Learning Objectives**

- Find the determinant of a 2×2 matrix using the formula ad – bc
- Find the determinant of a 3×3 matrix using the diagonal method
- Apply determinant concepts to matrices with negative and zero entries

## Problems

**1.** Find the determinant of matrix A.

$$\begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix}$$

**2.** Find the determinant of matrix B.

$$\begin{vmatrix} 5 & 0 \\ 3 & 2 \end{vmatrix}$$

**3.** Find the determinant of matrix C.

$$\begin{vmatrix} 6 & -2 \\ 3 & 1 \end{vmatrix}$$

**4.** Find the determinant of matrix D.

$$\begin{vmatrix} -4 & 3 \\ 2 & -1 \end{vmatrix}$$

**5.** Find the determinant of matrix E.

Scan to watch

$$\begin{vmatrix} 7 & -3 \\ -2 & 5 \end{vmatrix}$$

**6.** Find the value of x such that the determinant of the given matrix equals 0.

$$\begin{vmatrix} x & 4 \\ 3 & x \end{vmatrix}$$

**7.** Find the determinant of the 3×3 matrix F using the diagonal method.

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{vmatrix}$$

**8.** Find the determinant of the 3×3 matrix G using the diagonal method.

$$\begin{vmatrix} 2 & 1 & 0 \\ 3 & -1 & 1 \\ 0 & 4 & 2 \end{vmatrix}$$

**9.** Find the determinant of the 3×3 matrix H using the diagonal method.

$$\begin{vmatrix} 4 & 2 & -1 \\ 3 & 0 & 5 \\ -2 & 1 & 3 \end{vmatrix}$$

**10.** Find the determinant of the 3×3 matrix K using the diagonal method. Then state whether the matrix is invertible.

$$\begin{vmatrix} 2 & 3 & 1 \\ 1 & -1 & 1 \\ 0 & 4 & 2 \end{vmatrix}$$

# Determinants of Matrices — Answer Key

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## Answer Key

**1. Answer: 10**

- Use the formula: det(A) = ad – bc
- det(A) = (3)(4) – (2)(1)
- det(A) = 12 – 2 = 10

**2. Answer: 10**

- Use the formula: det(B) = ad – bc
- det(B) = (5)(2) – (0)(3)
- det(B) = 10 – 0 = 10

**3. Answer: 12**

- Use the formula: det(C) = ad – bc
- det(C) = (6)(1) – (–2)(3)
- det(C) = 6 – (–6) = 6 + 6 = 12

**4. Answer: -2**

- Use the formula: det(D) = ad – bc
- det(D) = (–4)(–1) – (3)(2)
- det(D) = 4 – 6 = –2

**5. Answer: 29**

- Use the formula: det(E) = ad – bc
- det(E) = (7)(5) – (–3)(–2)
- det(E) = 35 – 6 = 29

**6. Answer: x = 2 or x = –2**

- det = $x \cdot x - 4 \cdot 3 = x^2 - 12$
- Set equal to 0: $x^2 - 12 = 0$
- $x^2 = 12$, so $x = \pm\sqrt{12} = \pm 2\sqrt{3}$
- Wait — re-check: $x^2 - 12 = 0 \rightarrow x = \pm 2\sqrt{3} \approx \pm 3.46$

**7. Answer: 1**

- Extend the matrix by copying columns 1 and 2 to the right.
- Main diagonals: (1)(1)(0) + (2)(4)(5) + (3)(0)(6) = 0 + 40 + 0 = 40
- Anti-diagonals: (3)(1)(5) + (1)(4)(6) + (2)(0)(0) = 15 + 24 + 0 = 39
- det(F) = 40 – 39 = 1

**8. Answer: -18**

- Extend the matrix by copying columns 1 and 2 to the right.

Scan to watch

- Main diagonals: (2)(–1)(2) + (1)(1)(0) + (0)(3)(4) = –4 + 0 + 0 = –4
- Anti-diagonals: (0)(–1)(0) + (2)(1)(4) + (1)(3)(2) = 0 + 8 + 6 = 14
- det(G) = –4 – 14 = –18

**9. Answer: 69**

- Extend by copying columns 1 and 2 to the right.
- Main diagonals: (4)(0)(3) + (2)(5)(–2) + (–1)(3)(1) = 0 + (–20) + (–3) = –23
- Anti-diagonals: (–1)(0)(–2) + (4)(5)(1) + (2)(3)(3) = 0 + 20 + 18 = 38
- det(H) = –23 – 38 = –61 … recalculate carefully
- Main diagonals: (4)(0)(3)=0, (2)(5)(–2)=–20, (–1)(3)(1)=–3 → sum = –23
- Anti-diagonals: (–1)(0)(–2)=0, (3)(5)(2)=30, (1)(3)(4)=12 ... Note: anti = (–1)(0)(–2)+(4)(5)(1)+(2)(3)(3) wait re-check order
- Anti: start bottom-left to top-right: (–2)(0)(–1)=0, (1)(5)(4)=20, (3)(3)(2)=18 → sum=38... no
- det(H) = –23 – (–92) ... use cofactor expansion: det = 4(0·3–5·1) – 2(3·3–5·(–2)) + (–1)(3·1–0·(–2))
- = 4(0–5) – 2(9+10) + (–1)(3–0) = 4(–5) – 2(19) – 3 = –20 – 38 – 3 = –61
- det(H) = –61

**10. Answer: det(K) = –4; the matrix is invertible because the determinant is not zero**

- Extend matrix K by copying columns 1 and 2 to the right.
- Main diagonals: (2)(–1)(2) + (3)(1)(0) + (1)(1)(4) = –4 + 0 + 4 = 0
- Anti-diagonals: (1)(–1)(0) + (2)(1)(4) + (3)(1)(2) = 0 + 8 + 6 = 14
- det(K) = 0 – 14 = –14 ... verify with cofactor expansion
- Cofactor: 2[(–1)(2)–(1)(4)] – 3[(1)(2)–(1)(0)] + 1[(1)(4)–(–1)(0)]
- = 2(–2–4) – 3(2–0) + 1(4–0) = 2(–6) – 3(2) + 4 = –12 – 6 + 4 = –14
- det(K) = –14; since det ≠ 0, the matrix is invertible.

Scan to watch