Inverse of 2×2 Matrices

Linear Algebra Worksheet · Grade 10–12

Name: ________________________________ Date: ___________________

Learning Objectives

- Verify that two matrices are inverses by showing their product equals the identity matrix
- Apply the inverse formula to find the inverse of a 2×2 matrix using the determinant
- Identify when a 2×2 matrix is not invertible (singular matrix)

Problems

1. Identify the 2×2 identity matrix from the list below. Which of the following is the identity matrix?

2. Find the determinant of matrix A.

$$\begin{vmatrix} 4 & 2 \\ 3 & 1 \end{vmatrix}$$

3. Multiply matrix A times matrix B and verify that the result is the identity matrix.

$$\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$

4. Find the inverse of matrix A using the inverse formula.

$$\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

5. Find the inverse of matrix A using the inverse formula.

$$\begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$$

Scan to watch

6. Determine whether the matrix below is invertible. If it is not invertible, explain why.

$$\begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$$

7. Show that B is the inverse of A by proving that A times B equals the identity matrix AND B times A equals the identity matrix.

$$A = \begin{bmatrix}1 & 1\\-2 & -1\end{bmatrix}, \quad B = \begin{bmatrix}-1 & -1\\2 & 1\end{bmatrix}$$

8. Find the inverse of matrix A, then verify your answer by computing A times A-inverse.

$$\begin{bmatrix} 4 & 7 \\ 2 & 3 \end{bmatrix}$$

9. Matrix A is given below. Find A-inverse, then use it to solve the system of equations for x and y by computing A-inverse times the column vector [8, 5].

$$\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$$

10. Given that matrix A equals the one shown, find the value of k such that the matrix is NOT invertible (i.e., its determinant equals zero), then find the inverse of A when k equals 1.

$$\begin{bmatrix} k & 3 \\ 2 & 6 \end{bmatrix}$$

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Inverse of 2×2 Matrices — Answer Key

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Answer Key

1. Answer: The matrix with 1s on the main diagonal and 0s elsewhere

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- The identity matrix I has 1s along the main diagonal (top-left to bottom-right).
- All other entries are 0.
- So the 2×2 identity matrix is [[1, 0], [0, 1]].

2. Answer: det(A) = -2

- For a 2×2 matrix [[a,b],[c,d]], the determinant is ad - bc.
- det(A) = (4)(1) - (2)(3)
- det(A) = 4 - 6 = -2

3. Answer: A × B = I (identity matrix)

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Multiply row 1 of A by column 1 of B: (2)(3) + (1)(-5) = 6 - 5 = 1
- Multiply row 1 of A by column 2 of B: (2)(-1) + (1)(2) = -2 + 2 = 0
- Multiply row 2 of A by column 1 of B: (5)(3) + (3)(-5) = 15 - 15 = 0
- Multiply row 2 of A by column 2 of B: (5)(-1) + (3)(2) = -5 + 6 = 1
- Result: [[1, 0], [0, 1]] = I ✓

4. Answer: A inverse = [[1, -1], [-2, 3]]

$$\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$$

- Identify a=3, b=1, c=2, d=1.
- Calculate the determinant: det = ad - bc = (3)(1) - (1)(2) = 3 - 2 = 1.
- Apply the inverse formula: A^{-1} = (1/det) × [[d, -b], [-c, a]].
- A^{-1} = (1/1) × [[1, -1], [-2, 3]]
- A^{-1} = [[1, -1], [-2, 3]]

5. Answer: A inverse = [[-1, 2], [3, -5]]

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$$\begin{bmatrix} -1 & 2 \\ 3 & -5 \end{bmatrix}$$

- Identify a=5, b=2, c=3, d=1.
- Calculate the determinant: det = (5)(1) - (2)(3) = 5 - 6 = -1.
- Apply the inverse formula: A^{-1} = (1/-1) × [[1, -2], [-3, 5]].
- A^{-1} = [[-1, 2], [3, -5]]

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6. Answer: Not invertible; determinant = 0

- Calculate the determinant: det = (2)(2) - (4)(1) = 4 - 4 = 0.
- Because the determinant equals 0, the matrix is singular (not invertible).
- A matrix is only invertible when its determinant is non-zero.

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7. Answer: Both AB and BA equal the identity matrix, so B is the inverse of A.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Compute AB: row1×col1 = (1)(-1)+(1)(2)=1, row1×col2 = (1)(-1)+(1)(1)=0
- row2×col1 = (-2)(-1)+(-1)(2)=0, row2×col2 = (-2)(-1)+(-1)(1)=1 → AB = I ✓
- Compute BA: row1×col1 = (-1)(1)+(-1)(-2)=1, row1×col2 = (-1)(1)+(-1)(-1)=0
- row2×col1 = (2)(1)+(1)(-2)=0, row2×col2 = (2)(1)+(1)(-1)=1 → BA = I ✓
- Since AB = BA = I, B is the inverse of A.

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8. Answer: A inverse = [[-3, 7], [2, -4]]

$$\begin{bmatrix} -3 & 7 \\ 2 & -4 \end{bmatrix}$$

- Identify a=4, b=7, c=2, d=3.
- det = (4)(3) - (7)(2) = 12 - 14 = -2.
- A^{-1} = (1/-2) × [[3, -7], [-2, 4]] = [[-3/2, 7/2], [1, -2]].
- Verify: A × A^{-1} row1×col1 = (4)(-3/2)+(7)(1) = -6+7 = 1 ✓
- row1×col2 = (4)(7/2)+(7)(-2) = 14-14 = 0 ✓; similarly rows 2 gives [0,1] ✓

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9. Answer: x = 14, y = -17

- det = (3)(3) - (2)(4) = 9 - 8 = 1.
- A^{-1} = (1/1) × [[3, -2], [-4, 3]] = [[3, -2], [-4, 3]].
- Multiply A^{-1} by [8, 5]: x = (3)(8)+(-2)(5) = 24-10 = 14
- y = (-4)(8)+(3)(5) = -32+15 = -17
- Solution: x = 14, y = -17

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10. Answer: k = 1 makes det = 0... wait: not invertible when k = 1; inverse when k = 2 is (1/6)[[6,-3],[-2,2]]

- det = 6k - (3)(2) = 6k - 6.
- Set det = 0: 6k - 6 = 0 → k = 1. So when k=1 the matrix is NOT invertible.

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- Now find inverse when k = 2: matrix is [[2,3],[2,6]], det = (2)(6)-(3)(2) = 12-6 = 6.
- A^{-1} = (1/6) × [[6,-3],[-2,2]]
- A^{-1} = [[1, -1/2], [-1/3, 1/3]]

Scan to watch