Solving 3-Variable Linear Systems Using Determinants and Cramer's Rule

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Learning Objectives

- Compute the determinant of a 2x2 matrix
- Compute the determinant of a 3x3 matrix using expansion or the diagonal method
- Apply Cramer's Rule to solve a system of linear equations in three variables
- Recognize when Cramer's Rule fails (determinant equal to zero)

Evaluate each determinant or solve each linear system using Cramer's Rule, showing all coefficient and replacement matrices.

1. Find the determinant of the 2x2 matrix.

begin vmatrix 3 & 2 5 & 4 end vmatrix

Answer: ________________________________

2. Find the determinant of the 2x2 matrix.

begin vmatrix 6 & -1 2 & 3 end vmatrix

Answer: ________________________________

3. Find the determinant of the 3x3 matrix using cofactor expansion along the first row.

begin vmatrix 1 & 2 & 3 0 & 4 & 5 1 & 0 & 6 end vmatrix

Answer: ________________________________

4. Find the determinant of the 3x3 matrix using the diagonal (Sarrus) method.

begin vmatrix 2 & 1 & 3 1 & 0 & 2 4 & 1 & 1 end vmatrix

Answer: ________________________________

5. Evaluate the determinant.

begin vmatrix 1 & -2 & 0 3 & 1 & 4 -1 & 2 & 5 end vmatrix

Answer: ________________________________

6. Use Cramer's Rule to solve the system for x only. State only the value of x.

$$\begin{cases} x + y + z = 6 \\ 2x - y + z = 3 \\ x + 2y - z = 2 \end{cases}$$

Answer: ________________________________

7. Use Cramer's Rule to solve for y in the system from the previous problem.

$$\begin{cases} x+y+z=6 \\ 2x-y+z=3 \\ x+2y-z=2 \end{cases}$$

Answer: ______________________________

8. Use Cramer's Rule to solve for z in the same system.

$$\begin{cases} x+y+z=6 \\ 2x-y+z=3 \\ x+2y-z=2 \end{cases}$$

Answer: ______________________________

9. Use Cramer's Rule to solve the system completely for x, y, and z.

$$\begin{cases} 2x+y-z=3 \\ x-y+z=0 \\ 3x+2y+z=8 \end{cases}$$

Answer: ______________________________

10. Determine whether Cramer's Rule can be used to solve the system. If not, explain why.

$$\begin{cases} x+2y+3z=4 \\ 2x+4y+6z=8 \\ x-y+z=2 \end{cases}$$

Answer: ______________________________

This worksheet covers finding determinants of 2x2 and 3x3 matrices and applying Cramer's Rule to solve systems of linear equations with three variables (x, y, z), matching the chapters of the video.

Solutions

1. Find the determinant of the 2x2 matrix.

begin vmatrix 3 & 2 5 & 4 end vmatrix

→ *Multiply the entries on the main diagonal: three times four equals twelve.*
→ *Multiply the entries on the anti-diagonal: two times five equals ten.*
→ *Subtract the anti-diagonal product from the main-diagonal product: twelve minus ten equals two.*

Answer: 2

2. Find the determinant of the 2x2 matrix.

begin vmatrix 6 & -1 2 & 3 end vmatrix

→ *Multiply the main diagonal: six times three equals eighteen.*
→ *Multiply the anti-diagonal: negative one times two equals negative two.*
→ *Subtract: eighteen minus negative two equals twenty.*

Answer: 20

3. Find the determinant of the 3x3 matrix using cofactor expansion along the first row.

begin vmatrix 1 & 2 & 3 0 & 4 & 5 1 & 0 & 6 end vmatrix

→ *Expand along the first row using one, two, and three with alternating signs.*
→ *The first 2x2 minor is four times six minus five times zero, which equals twenty-four.*
→ *The second 2x2 minor is zero times six minus five times one, which equals negative five.*
→ *The third 2x2 minor is zero times zero minus four times one, which equals negative four.*
→ *Combine: one times twenty-four minus two times negative five plus three times negative four equals twenty-four plus ten minus twelve, which equals twenty-two.*

Answer: 22

4. Find the determinant of the 3x3 matrix using the diagonal (Sarrus) method.

begin vmatrix 2 & 1 & 3 1 & 0 & 2 4 & 1 & 1 end vmatrix

→ *Compute the three down-right diagonal products: two times zero times one is zero, one times two times four is eight, and three times one times one is three.*
→ *Add these: zero plus eight plus three equals eleven.*
→ *Compute the three up-right diagonal products: three times zero times four is zero, two times two times one is four, and one times one times one is one + 1 = two; correct sum is zero plus four plus two equals six.*
→ *Subtract the up-right total from the down-right total: eleven minus six equals five.*

Answer: 5

5. Evaluate the determinant.

begin vmatrix 1 & -2 & 0 3 & 1 & 4 -1 & 2 & 5 end vmatrix

→ *Expand along the first row.*
→ *The first minor is one times five minus four times two, which equals negative three.*
→ *The second minor is three times five minus four times negative one, which equals nineteen.*
→ *The third minor is three times two minus one times negative one, which equals seven.*
→ *Combine with signs: one times negative three minus negative two times nineteen plus zero times seven equals negative three plus thirty-eight, which equals thirty-five; recheck: 1(-3) + 2(19) + 0 = -3 + 38 = 35; the determinant equals thirty-five.*

Answer: 39

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6. Use Cramer's Rule to solve the system for x only. State only the value of x.

$$\begin{cases} x+y+z=6 \\ 2x-y+z=3 \\ x+2y-z=2 \end{cases}$$

→ *Form the coefficient matrix D and compute its determinant; D equals negative five.*
→ *Replace the x-column with the constants to form D sub x and compute its determinant; D sub x equals negative five.*
→ *Apply Cramer's Rule: x equals D sub x divided by D, which equals negative five divided by negative five, which equals one.*

Answer: $x=1$

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7. Use Cramer's Rule to solve for y in the system from the previous problem.

$$\begin{cases} x+y+z=6 \\ 2x-y+z=3 \\ x+2y-z=2 \end{cases}$$

→ *Use the same coefficient determinant D equal to negative five.*
→ *Replace the y-column with the constants to form D sub y and compute its determinant; D sub y equals negative ten.*
→ *Apply Cramer's Rule: y equals D sub y divided by D, which equals negative ten divided by negative five, which equals two.*

Answer: $y=2$

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8. Use Cramer's Rule to solve for z in the same system.

$$\begin{cases} x+y+z=6 \\ 2x-y+z=3 \\ x+2y-z=2 \end{cases}$$

→ *Use the same coefficient determinant D equal to negative five.*
→ *Replace the z-column with the constants to form D sub z and compute its determinant; D sub z equals negative fifteen.*
→ *Apply Cramer's Rule: z equals D sub z divided by D, which equals negative fifteen divided by negative five, which equals three.*

Answer: $z=3$

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9. Use Cramer's Rule to solve the system completely for x, y, and z.

$$\begin{cases} 2x+y-z=3 \\ x-y+z=0 \\ 3x+2y+z=8 \end{cases}$$

→ *Compute the coefficient determinant D; D equals negative seven.*
→ *Form D sub x by replacing the x-column with the constants and compute; D sub x equals negative seven, so x equals one.*
→ *Form D sub y by replacing the y-column with the constants and compute; D sub y equals negative fourteen, so y equals two.*
→ *Form D sub z by replacing the z-column with the constants and compute; D sub z equals negative seven, so z equals one.*

Answer: $x=1,\ y=2,\ z=1$

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10. Determine whether Cramer's Rule can be used to solve the system. If not, explain why.

$$\begin{cases} x + 2y + 3z = 4 \\ 2x + 4y + 6z = 8 \\ x - y + z = 2 \end{cases}$$

$\rightarrow$ *Compute the determinant of the coefficient matrix D.*

$\rightarrow$ *Notice that the second row is exactly twice the first row, so the rows are linearly dependent.*

$\rightarrow$ *Therefore the determinant D equals zero, which means Cramer's Rule cannot be applied because division by zero is undefined.*

Answer: Cramer's Rule fails; $D = 0$