Linear Systems in Matrix Form: Determinants, Cramer's Rule, and Gauss-Jor

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Learning Objectives

- Compute determinants of 2x2 and 3x3 matrices
- Solve linear systems using Cramer's Rule
- Solve linear systems using Gauss-Jordan Elimination
- Verify matrix solutions using an online matrix web application

Solve each problem by hand, then verify your answer using the matrix.reshish.com web application.

1. Find the determinant of the 2x2 matrix.

begin vmatrix 5 & 3 2 & 4 end vmatrix

Answer: ________________________________

2. Find the determinant of the 3x3 matrix using cofactor expansion along the first row.

begin vmatrix 1 & 2 & 3 0 & 4 & 5 1 & 0 & 6 end vmatrix

Answer: ________________________________

3. Write the following linear system in matrix form AX = B.

$$\begin{cases} 2x + 3y = 7 \\ 4x - y = 5 \end{cases}$$

Answer: ________________________________

4. Use Cramer's Rule to solve for x in the system.

$$\begin{cases} 3x + 2y = 16 \\ x - y = 2 \end{cases}$$

Answer: ________________________________

5. Use Cramer's Rule to solve for y in the same system from Problem 4.

$$\begin{cases} 3x + 2y = 16 \\ x - y = 2 \end{cases}$$

Answer: ________________________________

6. Use Cramer's Rule to solve the 3x3 system for x.

$$\begin{cases} x + y + z = 6 \\ 2x - y + z = 3 \\ x + 2y - z = 2 \end{cases}$$

Answer: ________________________________

7. Use Gauss-Jordan Elimination to solve the system.

$$\begin{cases} x + 2y = 5 \\ 3x - y = 1 \end{cases}$$

Answer: ______________________________________

8. Use Gauss-Jordan Elimination to find the reduced row echelon form, then state the solution.

$$\begin{cases} x + y + z = 3 \\ 2x + 3y + z = 6 \\ x - y + 2z = 2 \end{cases}$$

Answer: ______________________________________

9. Determine whether the matrix is invertible by computing its determinant.

begin vmatrix 2 & 4 1 & 2 end vmatrix

Answer: ______________________________________

10. Using the matrix web application at matrix.reshish.com, a student entered the system below and got the result x = 2, y = -1, z = 3. Verify by computing the determinant of the coefficient matrix and confirming it is nonzero.

$$\begin{cases} x + y + z = 4 \\ 2x - y + z = 8 \\ x + 2y - z = -3 \end{cases}$$

Answer: ______________________________________

This worksheet covers determinants of 2x2 and 3x3 matrices, Cramer's Rule for solving linear systems, Gauss-Jordan Elimination, writing systems in matrix form, and verifying solutions using a free online matrix calculator web application as demonstrated in the video.

Solutions

1. Find the determinant of the 2x2 matrix.

begin vmatrix 5 & 3 2 & 4 end vmatrix

→ *Multiply the top-left entry by the bottom-right entry to get 5 times 4 equals 20.*
→ *Multiply the top-right entry by the bottom-left entry to get 3 times 2 equals 6.*
→ *Subtract the second product from the first to get 20 minus 6 equals 14.*

Answer: 14

2. Find the determinant of the 3x3 matrix using cofactor expansion along the first row.

begin vmatrix 1 & 2 & 3 0 & 4 & 5 1 & 0 & 6 end vmatrix

→ *Expand along the first row using cofactors.*
→ *Compute the first minor: 4 times 6 minus 5 times 0 equals 24, multiplied by 1 gives 24.*
→ *Compute the second minor: 0 times 6 minus 5 times 1 equals negative 5, multiplied by negative 2 gives 10.*
→ *Compute the third minor: 0 times 0 minus 4 times 1 equals negative 4, multiplied by 3 gives negative 12.*
→ *Add the cofactor products to get 24 plus 10 minus 12 equals 22.*

Answer: 22

3. Write the following linear system in matrix form AX = B.

$$\begin{cases} 2x + 3y = 7 \\ 4x - y = 5 \end{cases}$$

→ *Apply the elimination method to the system [2x + 3y = 7] and [4x - y = 5].*
→ *Reduce to one variable and solve to get x = 11/7.*
→ *Substitute back to get y = 9/7.*
→ *The solution is (11/7, 9/7); it checks in both original equations.*

Answer: $\left(\frac{11}{7}, \frac{9}{7}\right)$

4. Use Cramer's Rule to solve for x in the system.

$$\begin{cases} 3x + 2y = 16 \\ x - y = 2 \end{cases}$$

→ *Apply the elimination method to the system [3x + 2y = 16] and [x - y = 2].*
→ *Reduce to one variable and solve to get x = 4.*
→ *Substitute back to get y = 2.*
→ *The solution is (4, 2); it checks in both original equations.*

Answer: $(4, 2)$

5. Use Cramer's Rule to solve for y in the same system from Problem 4.

$$\begin{cases} 3x + 2y = 16 \\ x - y = 2 \end{cases}$$

→ *Apply the elimination method to the system [3x + 2y = 16] and [x - y = 2].*
→ *Reduce to one variable and solve to get x = 4.*
→ *Substitute back to get y = 2.*
→ *The solution is (4, 2); it checks in both original equations.*

Answer: $(4,\ 2)$

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6. Use Cramer's Rule to solve the 3x3 system for x.

$$\begin{cases} x + y + z = 6 \\ 2x - y + z = 3 \\ x + 2y - z = 2 \end{cases}$$

→ *Compute the determinant of the coefficient matrix and obtain 5.*
→ *Replace the x-column with the constants 6, 3, and 2, and compute its determinant to get 5.*
→ *Divide the x-determinant by the coefficient determinant: 5 divided by 5 equals 1.*

Answer: $x = 1$

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7. Use Gauss-Jordan Elimination to solve the system.

$$\begin{cases} x + 2y = 5 \\ 3x - y = 1 \end{cases}$$

→ *Apply the elimination method to the system [x + 2y = 5] and [3x - y = 1].*
→ *Reduce to one variable and solve to get x = 1.*
→ *Substitute back to get y = 2.*
→ *The solution is (1, 2); it checks in both original equations.*

Answer: $(1,\ 2)$

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8. Use Gauss-Jordan Elimination to find the reduced row echelon form, then state the solution.

$$\begin{cases} x + y + z = 3 \\ 2x + 3y + z = 6 \\ x - y + 2z = 2 \end{cases}$$

→ *Write the augmented matrix from the coefficients and constants.*
→ *Eliminate the x-terms below row 1 by subtracting suitable multiples of row 1.*
→ *Use row 2 to eliminate the y-term in row 3, then scale to make the leading entry 1.*
→ *Back-substitute by clearing entries above each leading 1 until the matrix is the identity on the left side.*
→ *Read off the solution as x equals 1, y equals 1, and z equals 1.*

Answer: $x = 1,\ y = 1,\ z = 1$

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9. Determine whether the matrix is invertible by computing its determinant.

begin vmatrix 2 & 4   1 & 2 end vmatrix

→ *Multiply 2 times 2 to get 4.*
→ *Multiply 4 times 1 to get 4.*
→ *Subtract to get 4 minus 4 equals 0.*
→ *Since the determinant is 0, the matrix is singular and not invertible, so Cramer's Rule cannot be used.*

Answer: 0, not invertible

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10. Using the matrix web application at matrix.reshish.com, a student entered the system below and got the result x = 2, y = -1, z = 3. Verify by computing the determinant of the coefficient matrix and confirming it is nonzero.

$$\begin{cases} x + y + z = 4 \\ 2x - y + z = 8 \\ x + 2y - z = -3 \end{cases}$$

→ *Write out the 3x3 coefficient matrix from the system.*

→ *Expand the determinant along the first row using cofactors.*

→ *Compute the three 2x2 minors and combine: 1 times negative 1 minus 2 minus 1 times negative 2 minus 1 plus 1 times 4 plus 1.*

→ *Simplify to obtain a determinant of 5, which is nonzero, confirming the system has a unique solution and the web application's result is valid.*

Answer: $\det(A) = 5$