Joint Variation

Algebra Worksheet · Grade 9–11

Name: _____ Date: _____

Learning Objectives

- Identify joint variation relationships and write the general equation y = kxz
- Find the constant of variation k using given values of the variables
- Solve real-world and multi-step problems involving joint variation

Problems

1. y varies jointly as x and z. Write the general equation that models this relationship.

$$y = kxz$$

2. y varies jointly as x and z. If y = 45 when x = 18 and z = 10, find the constant of variation k.

$$y = kxz$$

3. y varies jointly as x and z. Given y = 45 when x = 18 and z = 10, find the equation of variation.

$$y = kxz$$

4. Using the equation of variation found in Problem 3, find y when x = 20 and z = 30.

$$y = \frac{1}{4}xz$$

5. The area A of a rectangle varies jointly as its length l and width w. If the area is 84 square units when the length is 12 units and the width is 7 units, find the constant of variation and write the equation of variation.

$$A = klw$$

6. The force F on an object varies jointly as its mass m and acceleration a. If F = 200 Newtons when m = 25 kg and a = 8 m/s squared, find the equation of variation and then find F when m = 40 kg and a = 5 m/s

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squared.

$$F = kma$$

7. y varies jointly as x squared and z. If y = 72 when x = 3 and z = 8, find the constant of variation k, write the equation of variation, and then find y when x = 5 and z = 4.

$$y = kx^2z$$

8. The cost C of a plane ticket varies jointly as the number of layovers n and the distance d of the trip in miles. If a ticket costs $480 for a trip with 2 layovers over a distance of 800 miles, find the equation of variation. Then find the cost for a trip with 3 layovers over a distance of 1200 miles.

$$C = knd$$

9. w varies jointly as x, y, and z. If w = 210 when x = 3, y = 5, and z = 14, find the constant of variation k and the equation of variation. Then find w when x = 6, y = 4, and z = 10.

$$w = kxyz$$

10. The volume V of a cone varies jointly as its height h and the square of its base radius r. A cone with height 9 cm and base radius 4 cm has a volume of 48π cubic centimeters. Find the constant of variation k and write the equation of variation. Then find the volume of a cone with height 12 cm and base radius 5 cm. Leave your answer in terms of pi.

$$V = khr^2$$

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Joint Variation — Answer Key

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Answer Key

1. Answer: y = kxz, where k is the constant of variation

- Joint variation means y is proportional to the product of two or more variables.
- The general formula is y = kxz, where k is the constant of variation.

2. Answer: k = 1/4

- Start with the formula: y = kxz.
- Substitute y = 45, x = 18, z = 10: 45 = k(18)(10).
- Simplify: 45 = 180k.
- Divide both sides by 180: k = 45/180 = 1/4.

3. Answer: y = (1/4)xz

- Find k first: 45 = k(18)(10) → 45 = 180k → k = 1/4.
- Substitute k back into y = kxz.
- The equation of variation is y = (1/4)xz.

4. Answer: y = 150

- Use the equation y = (1/4)xz.
- Substitute x = 20 and z = 30: y = (1/4)(20)(30).
- Calculate: y = (1/4)(600) = 150.

5. Answer: k = 1, A = lw

- Write the joint variation equation: A = klw.
- Substitute A = 84, l = 12, w = 7: 84 = k(12)(7).
- Simplify: 84 = 84k.
- Divide: k = 1, so the equation is A = lw.

6. Answer: k = 1, F = ma; F = 200 Newtons

- Write the joint variation equation: F = kma.
- Substitute F = 200, m = 25, a = 8: 200 = k(25)(8) = 200k → k = 1.
- The equation is F = ma.
- Find F when m = 40, a = 5: F = (40)(5) = 200 Newtons.

7. Answer: k = 1; y = x^2z; y = 100

- Write the equation: y = kx^2z.
- Substitute y = 72, x = 3, z = 8: 72 = k(9)(8) = 72k → k = 1.
- The equation is y = x^2z.
- Find y when x = 5, z = 4: y = (25)(4) = 100.

8. Answer: k = 0.3; C = 0.3nd; C = $1080

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- Write the joint variation equation: $C = knd$.
- Substitute $C = 480$, $n = 2$, $d = 800$: $480 = k(2)(800) = 1600k$.
- Divide: $k = 480/1600 = 0.3$.
- Equation: $C = 0.3nd$.
- Find C when $n = 3$, $d = 1200$: $C = 0.3(3)(1200) = 0.3(3600) = \$1080$.

9. Answer: $k = 1$; $w = xyz$; $w = 240$

- Write the equation: $w = kxyz$.
- Substitute $w = 210$, $x = 3$, $y = 5$, $z = 14$: $210 = k(3)(5)(14) = 210k \rightarrow k = 1$.
- Equation: $w = xyz$.
- Find w when $x = 6$, $y = 4$, $z = 10$: $w = (6)(4)(10) = 240$.

10. Answer: $k = \pi/3$; $V = (\pi/3)hr^2$; $V = 100\pi$ cm³

- Write the joint variation equation: $V = khr^2$.
- Substitute $V = 48\pi$, $h = 9$, $r = 4$: $48\pi = k(9)(16) = 144k$.
- Solve for k: $k = 48\pi/144 = \pi/3$.
- Equation: $V = (\pi/3)hr^2$.
- Find V when $h = 12$, $r = 5$: $V = (\pi/3)(12)(25) = (\pi/3)(300) = 100\pi$ cm³.

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