Remainder Theorem & Factor Theorem

Algebra Worksheet · Grade 9–11

Name: ________________________________ Date: ___________________

Learning Objectives

- Use the Remainder Theorem to evaluate a polynomial function at a given value using synthetic division
- Apply the Factor Theorem to determine whether a given binomial is a factor of a polynomial
- Find all rational roots of a polynomial using the Rational Root Theorem and synthetic division

Problems

1. Use the Remainder Theorem and synthetic division to find f(2):

$$f(x) = x^3 - 4x^2 + 5x + 3$$

2. Use the Remainder Theorem and synthetic division to find f(2):

$$f(x) = -x^3 + 6x - 7$$

3. Use the Remainder Theorem to find f(-3):

$$f(x) = 2x^3 + x^2 - 5x + 4$$

4. Use the Remainder Theorem to find f(4):

$$f(x) = x^3 - 6x^2 + 3x - 2$$

5. Use the Factor Theorem to determine whether (x + 1) is a factor of the polynomial below. Show your work using synthetic division.

$$f(x) = x^3 + 9x^2 + 23x + 15$$

6. Use the Factor Theorem to determine whether (x - 3) is a factor of the polynomial below.

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$$f(x) = x^3 - 5x^2 + 2x + 8$$

7. List all possible rational roots of the polynomial below using the Rational Root Theorem, then find one actual root using synthetic division.

$$f(x) = x^3 + 2x^2 - 5x - 6$$

8. Find all real roots of the polynomial below using the Rational Root Theorem and synthetic division.

$$f(x) = x^3 + 9x^2 + 23x + 15$$

9. Find all real roots of the polynomial below using the Rational Root Theorem and synthetic division.

$$f(x) = 2x^3 - 3x^2 - 11x + 6$$

10. A polynomial f(x) of degree 3 has leading coefficient 1 and constant term -12. Using the Rational Root Theorem and synthetic division, find all real roots of the polynomial below and write it in fully factored form.

$$f(x) = x^3 - x^2 - 8x + 12$$

Remainder Theorem & Factor Theorem — Answer Key

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Answer Key

1. Answer: f(2) = 5

- Set up synthetic division with divisor 2 and coefficients 1, -4, 5, 3
- Bring down 1; multiply 1×2=2, add to -4 to get -2
- Multiply -2×2=-4, add to 5 to get 1
- Multiply 1×2=2, add to 3 to get remainder 5
- By the Remainder Theorem, f(2) = 5

2. Answer: f(2) = -3

- Rewrite with all terms: $-x^3 + 0x^2 + 6x - 7$, so coefficients are -1, 0, 6, -7
- Set up synthetic division with divisor 2
- Bring down -1; multiply -1×2=-2, add to 0 to get -2
- Multiply -2×2=-4, add to 6 to get 2
- Multiply 2×2=4, add to -7 to get remainder -3
- By the Remainder Theorem, f(2) = -3

3. Answer: f(-3) = -26

- Coefficients are 2, 1, -5, 4; divisor is -3
- Bring down 2; multiply 2×(-3)=-6, add to 1 to get -5
- Multiply -5×(-3)=15, add to -5 to get 10
- Multiply 10×(-3)=-30, add to 4 to get remainder -26
- By the Remainder Theorem, f(-3) = -26

4. Answer: f(4) = -18

- Coefficients are 1, -6, 3, -2; divisor is 4
- Bring down 1; multiply 1×4=4, add to -6 to get -2
- Multiply -2×4=-8, add to 3 to get -5
- Multiply -5×4=-20, add to -2 to get remainder -22
- By the Remainder Theorem, f(4) = -22

5. Answer: Yes, (x + 1) is a factor because f(-1) = 0

- If (x + 1) is a factor, then f(-1) must equal 0
- Coefficients are 1, 9, 23, 15; divisor is -1
- Bring down 1; multiply 1×(-1)=-1, add to 9 to get 8
- Multiply 8×(-1)=-8, add to 23 to get 15
- Multiply 15×(-1)=-15, add to 15 to get remainder 0
- Remainder = 0, so (x + 1) is a factor

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6. Answer: No, $(x - 3)$ is NOT a factor because $f(3) = -4$

- If $(x - 3)$ is a factor, then $f(3)$ must equal 0
- Coefficients are 1, -5, 2, 8; divisor is 3
- Bring down 1; multiply $1 \times 3 = 3$, add to -5 to get -2
- Multiply $-2 \times 3 = -6$, add to 2 to get -4
- Multiply $-4 \times 3 = -12$, add to 8 to get remainder -4
- Remainder $\neq 0$, so $(x - 3)$ is NOT a factor

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7. Answer: Possible roots: $\pm 1, \pm 2, \pm 3, \pm 6$; one root is $x = 2$

- Factors of leading coefficient (1): ± 1
- Factors of constant (-6): $\pm 1, \pm 2, \pm 3, \pm 6$
- Possible rational roots: $\pm 1, \pm 2, \pm 3, \pm 6$
- Test $x = 2$ with coefficients 1, 2, -5, -6
- Bring down 1; $1 \times 2 = 2$, add to 2 = 4; $4 \times 2 = 8$, add to -5 = 3; $3 \times 2 = 6$, add to -6 = 0
- Remainder = 0, so $x = 2$ is a root

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8. Answer: $x = -1$, $x = -3$, $x = -5$

- Possible rational roots from factors of 15 over factors of 1: $\pm 1, \pm 3, \pm 5, \pm 15$
- Test $x = -1$: remainder = 0, so $(x + 1)$ is a factor; quotient is $x^2 + 8x + 15$
- Factor $x^2 + 8x + 15 = (x + 3)(x + 5)$
- Set each factor to zero: $x = -1$, $x = -3$, $x = -5$

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9. Answer: $x = 3$, $x = -2$, $x = 1/2$

- Possible rational roots: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 1/2, \pm 3/2$
- Test $x = 3$ with coefficients 2, -3, -11, 6: remainder = 0; quotient is $2x^2 + 3x - 2$
- Factor $2x^2 + 3x - 2 = (2x - 1)(x + 2)$
- Set each factor to zero: $x = 3$, $x = 1/2$, $x = -2$

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10. Answer: $x = 2$ (double root), $x = -3$; factored form: $(x - 2)^2(x + 3)$

- Possible rational roots from factors of 12: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$
- Test $x = 2$ with coefficients 1, -1, -8, 12: remainder = 0; quotient is $x^2 + x - 6$
- Factor $x^2 + x - 6 = (x - 2)(x + 3)$
- So $f(x) = (x - 2)(x - 2)(x + 3) = (x - 2)^2(x + 3)$
- Roots are $x = 2$ (multiplicity 2) and $x = -3$
