Asymptotes of Rational Functions

Algebra 2 Worksheet · Grade 10–12

Name: ______________________________ Date: __________________

Learning Objectives

- Find vertical asymptotes of rational functions by setting the denominator equal to zero and solving
- Determine horizontal asymptotes by comparing the degrees of the numerator and denominator
- Identify when no horizontal asymptote exists based on the relationship between exponents

Problems

1. Find the vertical asymptote of the rational function below.

$$f(x) = \frac{3}{x-5}$$

2. Find the vertical asymptote of the rational function below.

$$f(x) = \frac{x+2}{x+7}$$

3. Determine the horizontal asymptote of the rational function below. Compare the degree of the numerator to the degree of the denominator.

$$f(x) = \frac{4x}{x^3+1}$$

4. Find both vertical asymptotes of the rational function below.

$$f(x) = \frac{2x^2}{x^2-1}$$

5. Determine the horizontal asymptote of the rational function below.

$$f(x) = \frac{3x^2}{x^2-4}$$

Scan to watch

6. Find both vertical asymptotes of the rational function below.

$$f(x) = \frac{x^2 + x - 2}{x^2 - x - 6}$$

7. Determine the horizontal asymptote of the rational function below. State whether no horizontal asymptote exists, y equals zero, or give the exact asymptote.

$$f(x) = \frac{x^3 + 2x}{x^2 - 5}$$

8. Find the vertical asymptotes and state the horizontal asymptote of the rational function below.

$$f(x) = \frac{5x}{x^2 - 9}$$

9. Find all vertical asymptotes of the rational function below. Hint: factor both the numerator and denominator first to check for common factors.

$$f(x) = \frac{x^2 - 4}{x^2 + x - 6}$$

10. For the rational function below, find all vertical asymptotes, state the horizontal asymptote, and explain your reasoning for the horizontal asymptote based on the degrees of the numerator and denominator.

$$f(x) = \frac{2x^2 + 3x - 2}{x^2 - 5x + 6}$$

Scan to watch

Asymptotes of Rational Functions — Answer Key

Algebra 2 Worksheet · Grade 10–12

Answer Key

1. Answer: x = 5
- Set the denominator equal to zero: x - 5 = 0
- Solve for x: x = 5

2. Answer: x = -7
- Set the denominator equal to zero: x + 7 = 0
- Solve for x: x = -7

3. Answer: y = 0
- Degree of numerator = 1, degree of denominator = 3
- Since degree of numerator is less than degree of denominator, the horizontal asymptote is y = 0

4. Answer: x = -1 and x = 1
- Set denominator equal to zero: $x^2 - 1 = 0$
- Factor using difference of squares: (x + 1)(x - 1) = 0
- Apply zero product property: x = -1 or x = 1

5. Answer: y = 3
- Degree of numerator = 2, degree of denominator = 2 (degrees are equal)
- When degrees are equal, the horizontal asymptote is the ratio of leading coefficients: y = 3/1 = 3

6. Answer: x = -2 and x = 3
- Set denominator equal to zero: $x^2 - x - 6 = 0$
- Factor: (x + 2)(x - 3) = 0
- Apply zero product property: x = -2 or x = 3

7. Answer: No horizontal asymptote
- Degree of numerator = 3, degree of denominator = 2
- Since degree of numerator is greater than degree of denominator, there is no horizontal asymptote

8. Answer: Vertical: x = -3 and x = 3; Horizontal: y = 0
- Set denominator equal to zero: $x^2 - 9 = 0$
- Factor: (x + 3)(x - 3) = 0, so x = -3 or x = 3
- Degree of numerator (1) is less than degree of denominator (2), so horizontal asymptote is y = 0

9. Answer: x = -3 only (x = 2 is a hole, not an asymptote)
- Factor numerator: (x + 2)(x - 2)
- Factor denominator: (x + 3)(x - 2)
- Cancel the common factor (x - 2); x = 2 is a hole, not a vertical asymptote

Scan to watch

- Remaining denominator factor gives vertical asymptote at $x = -3$

10. Answer: Vertical: x = 2 and x = 3; Horizontal: y = 2

- Set denominator equal to zero: $x^2 - 5x + 6 = 0$
- Factor: $(x - 2)(x - 3) = 0$, so $x = 2$ or $x = 3$
- Check numerator at $x = 2$: $2(4) + 3(2) - 2 = 12$, not zero, so $x = 2$ is a vertical asymptote
- Check numerator at $x = 3$: $2(9) + 3(3) - 2 = 25$, not zero, so $x = 3$ is a vertical asymptote
- Degree of numerator = 2, degree of denominator = 2 (equal degrees)
- Horizontal asymptote is the ratio of leading coefficients: $y = 2/1 = 2$