Graphing Logarithmic Functions

Algebra 2 / Pre-Calculus Worksheet · Grade 9–11

Name: ______________________________ Date: __________________

Learning Objectives

- Identify the domain, range, and intercepts of a logarithmic function
- Understand the relationship between exponential and logarithmic graphs as inverses reflected over y = x
- Describe and sketch the behavior of logarithmic functions with varying bases

Problems

1. State the x-intercept of the basic logarithmic function shown below:

$$y = \log_a x$$

2. State the domain and range of the logarithmic function below:

$$y = \log_5 x$$

3. The exponential function below has a y-intercept at (0, 1). What is the x-intercept of its inverse logarithmic function?

$$y = a^x$$

4. Convert the exponential equation below into its equivalent logarithmic form:

$$y = 3^x$$

5. Both functions below are logarithmic. Do they share the same general graph shape and x-intercept? Explain briefly and state the shared x-intercept.

$$f(x) = \log_5 x \quad \text{and} \quad g(x) = \log_{32} x$$

Scan to watch

6. The graph of a logarithmic function is a reflection of its corresponding exponential function over a specific line. Identify that line and write its equation.

7. Evaluate the logarithmic expression below without a calculator:

$$\log_2 8$$

8. Describe how the graph of the function below compares to the graph of y = log_7(x). Then state the domain and range.

$$y = \log_7(-x)$$

9. Find the inverse of the logarithmic function below, then state the domain and range of the inverse:

$$f(x) = \log_4 x$$

10. Determine the domain, range, x-intercept, and describe the transformation applied to the parent logarithmic function for the function below:

$$y = \log_3(x - 2) + 1$$

Scan to watch

Graphing Logarithmic Functions — Answer Key

Algebra 2 / Pre-Calculus Worksheet · Grade 9–11

Answer Key

1. Answer: (1, 0)

- Set y = 0: 0 = log_a(x)
- Convert to exponential form: a^0 = x, so x = 1
- The x-intercept is (1, 0)

2. Answer: Domain: x > 0; Range: all real numbers

- The logarithm is only defined for positive values of x
- Domain: x > 0 (or (0, ∞))
- The output y can be any real number, so Range: all real numbers (or (-∞, ∞))

3. Answer: (1, 0)

- The inverse of y = a^x is y = log_a(x)
- Because the inverse swaps x and y coordinates, the y-intercept (0,1) of the exponential becomes the x-intercept (1,0) of the logarithm

4. Answer: x = log base 3 of y, or log_3(y) = x

- The exponential form y = 3^x means '3 raised to x equals y'
- Converting: x = log_3(y)
- This is the inverse (logarithmic) form of the original equation

5. Answer: Yes, both have the same general shape and share x-intercept (1, 0)

- Both functions are of the form y = log_b(x) with a positive base
- All such logarithmic functions pass through (1, 0) since log_b(1) = 0 for any valid base b
- The general shape (increasing curve, domain x > 0) is the same regardless of the specific base

6. Answer: y = x

- Exponential and logarithmic functions are inverses of each other
- Inverse functions are reflections of each other over the line y = x
- Therefore the line of reflection is y = x

7. Answer: 3

- Ask: 2 raised to what power equals 8?
- 2^3 = 8
- Therefore log_2(8) = 3

8. Answer: Reflected over the y-axis; Domain: x < 0; Range: all real numbers

- Replacing x with -x reflects the graph over the y-axis
- For log_7(-x) to be defined, we need -x > 0, which means x < 0
- Domain: x < 0; Range: all real numbers (unchanged by reflection over y-axis)

Scan to watch

9. Answer: Inverse: f inverse(x) = 4^x; Domain of inverse: all real numbers; Range of inverse: y > 0

- Replace f(x) with y: y = log_4(x)
- Swap x and y: x = log_4(y)
- Convert to exponential form: y = 4^x
- The inverse is f^{-1}(x) = 4^x, with domain all real numbers and range y > 0

---

10. Answer: Domain: x > 2; Range: all real numbers; x-intercept: (3 - 10^0... specifically x = 2 + 3^{-1} = 2 + 1/3... solve: x-intercept at x = 2 + 1/3 ≈ 2.33); shifted right 2 and up 1

- Parent function is y = log_3(x), which has domain x > 0
- Replacing x with (x - 2) shifts the graph 2 units to the right, so domain becomes x - 2 > 0, meaning x > 2
- Adding 1 shifts the graph 1 unit upward; Range remains all real numbers
- To find x-intercept, set y = 0: 0 = log_3(x-2) + 1 → log_3(x-2) = -1 → x - 2 = 3^{-1} = 1/3 → x = 2 + 1/3 = 7/3
- X-intercept: (7/3, 0); the graph is the parent log_3(x) shifted right 2 units and up 1 unit

---

Scan to watch