Solving Logarithmic Equations

Algebra Worksheet · Grade 10–12

Name: _____ Date: _____

Learning Objectives

- Apply logarithm properties (power, product, quotient) to simplify logarithmic expressions
- Solve logarithmic equations by canceling logarithms when both sides share the same base
- Convert between logarithmic and exponential form to solve equations leading to linear or quadratic solutions

Problems

1. Solve the logarithmic equation by canceling the logarithms on both sides.

$$\log_b(x+3) = \log_b(7)$$

2. Solve the logarithmic equation by canceling the logarithms on both sides.

$$\log_5(2x-1) = \log_5(9)$$

3. Use the power property of logarithms to rewrite the left side, then solve.

$$3\log_b(x) = \log_b(27)$$

4. Use the product property of logarithms to combine the left side, then convert to exponential form and solve.

$$\log_2(x) + \log_2(4) = 3$$

5. Solve the equation. Use the cancellation property after matching both sides.

$$\log_b(3x+4) = \log_b(5x-2)$$

6. Use the product property to combine the left side, convert to exponential form, and solve the resulting equation.

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$$\log_3(x) + \log_3(x-2) = 1$$

7. Use the power property on the left side and the product property on the right side, then solve.

$$2\log_b(x) = \log_b(3) + \log_b(x+6)$$

8. Combine using the quotient property on the left side, convert to exponential form, and solve the quadratic.

$$\log_2(x+5) - \log_2(x-1) = 2$$

9. Use logarithm properties to simplify both sides, then solve the resulting quadratic equation.

$$2\log_b(x) = \log_b(4) + \log_b(x-1)$$

10. Simplify using logarithm properties on both sides, convert to exponential form if needed, and solve the resulting quadratic equation.

$$\log_2(x) + \log_2(x-2) = \log_2(x+4)$$

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Solving Logarithmic Equations — Answer Key

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Answer Key

1. Answer: x = 4
- Since both sides have the same base b, cancel the logarithms: x + 3 = 7
- Subtract 3 from both sides: x = 4

2. Answer: x = 5
- Cancel the logarithms (same base 5): 2x - 1 = 9
- Add 1 to both sides: 2x = 10, then divide by 2: x = 5

3. Answer: x = 3
- Apply power property: 3 log_b(x) = log_b(x^3)
- Cancel logarithms: x^3 = 27
- Take the cube root: x = 3

4. Answer: x = 2
- Apply product property: log_2(4x) = 3
- Convert to exponential form: 4x = 2^3 = 8
- Divide both sides by 4: x = 2

5. Answer: x = 3
- Cancel the logarithms (same base): 3x + 4 = 5x - 2
- Subtract 3x from both sides: 4 = 2x - 2
- Add 2: 6 = 2x, so x = 3

6. Answer: x = 3
- Apply product property: log_3(x(x-2)) = 1
- Convert to exponential: x(x-2) = 3^1 = 3
- Expand: x^2 - 2x - 3 = 0
- Factor: (x-3)(x+1) = 0, so x = 3 or x = -1
- Reject x = -1 (negative argument), so x = 3

7. Answer: x = 6
- Power property: log_b(x^2) = log_b(3(x+6))
- Cancel logarithms: x^2 = 3x + 18
- Rearrange: x^2 - 3x - 18 = 0
- Factor: (x-6)(x+3) = 0, so x = 6 or x = -3
- Reject x = -3 (negative argument), so x = 6

8. Answer: x = 3
- Apply quotient property: log_2((x+5)/(x-1)) = 2

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- Convert to exponential: (x+5)/(x-1) = 4
- Multiply both sides by (x-1): x + 5 = 4x - 4
- Solve: 9 = 3x, so x = 3
- Check: x = 3 gives valid (positive) arguments

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9. Answer: x = 2

- Power property left side: log_b(x^2); product property right side: log_b(4(x-1))
- Cancel logarithms: x^2 = 4x - 4
- Rearrange: x^2 - 4x + 4 = 0
- Factor: (x-2)^2 = 0, so x = 2
- Check: x = 2 gives valid arguments in original equation

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10. Answer: x = 4

- Product property on left: log_2(x(x-2)) = log_2(x+4)
- Cancel logarithms: x(x-2) = x + 4
- Expand: x^2 - 2x = x + 4
- Rearrange: x^2 - 3x - 4 = 0
- Factor: (x-4)(x+1) = 0, so x = 4 or x = -1
- Reject x = -1 (negative argument), so x = 4

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