Unit Circle & Special Right Triangles

Trigonometry Worksheet · Grade 10–12

Name: ________________________________ Date: ___________________

Learning Objectives

- Identify side lengths of 30-60-90 and 45-45-90 special right triangles
- Determine the coordinates (cosine, sine) of key angles on the unit circle
- Apply quadrant sign rules to find exact trigonometric values

Problems

1. In a 45-45-90 special right triangle inscribed in the unit circle, what is the length of each leg?

2. In a 30-60-90 special right triangle inscribed in the unit circle, what is the length of the shorter leg?

3. What are the coordinates (cosine, sine) of the point on the unit circle at 45 degrees?

$$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

4. What are the coordinates (cosine, sine) of the point on the unit circle at 30 degrees?

$$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

5. What are the coordinates (cosine, sine) of the point on the unit circle at 60 degrees?

$$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

6. A point on the unit circle is in the second quadrant at 135 degrees. Using the 45-45-90 reference triangle, state the sign of cosine and sine, then write the exact coordinates.

$$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

Scan to watch

7. Find the exact coordinates of the point on the unit circle at 210 degrees using the 30-60-90 reference triangle and quadrant sign rules.

$$\left(-\frac{\sqrt{3}}{2},\ -\frac{1}{2}\right)$$

8. Find the exact value of sine of 315 degrees using the unit circle and special right triangle relationships.

$$\sin(315^\circ)$$

9. Using the unit circle, evaluate the expression cosine squared of 30 degrees plus sine squared of 30 degrees and explain what identity this demonstrates.

$$\cos^2(30^\circ) + \sin^2(30^\circ)$$

10. A point P on the unit circle has a cosine value of negative one-half. List ALL angles between 0 and 360 degrees where this occurs, state the sine value at each angle, and identify which quadrants they lie in.

$$\cos(\theta) = -\frac{1}{2}, \quad 0^\circ \leq \theta < 360^\circ$$

Scan to watch

Unit Circle & Special Right Triangles — Answer Key

Trigonometry Worksheet · Grade 10–12

Answer Key

1. Answer: √2 / 2

- The hypotenuse of the triangle equals the radius of the unit circle, which is 1.
- For a 45-45-90 triangle, each leg equals hypotenuse ÷ √2 = 1 ÷ √2 = √2 / 2.

2. Answer: 1/2

- The hypotenuse equals the radius of the unit circle, which is 1.
- In a 30-60-90 triangle, the shorter leg (opposite 30°) = hypotenuse / 2 = 1/2.

3. Answer: (√2/2, √2/2)

- Use the 45-45-90 triangle: both legs equal √2/2.
- The horizontal leg gives cosine(45°) = √2/2 and the vertical leg gives sine(45°) = √2/2.

4. Answer: (√3/2, 1/2)

- Use the 30-60-90 triangle: longer leg = √3/2, shorter leg = 1/2.
- The horizontal leg (adjacent to 30°) gives cosine(30°) = √3/2; the vertical leg gives sine(30°) = 1/2.

5. Answer: (1/2, √3/2)

- Use the 30-60-90 triangle rotated so the 60° angle is at the origin.
- The horizontal leg (adjacent to 60°) = 1/2 and the vertical leg (opposite 60°) = √3/2.

6. Answer: (-√2/2, √2/2)

- The reference angle for 135° is 45°, so the magnitudes are √2/2 for both values.
- In Quadrant II, cosine is negative and sine is positive, giving (-√2/2, √2/2).

7. Answer: (-√3/2, -1/2)

- 210° lies in Quadrant III; the reference angle is 210° - 180° = 30°.
- From the 30-60-90 triangle: cosine(30°) = √3/2, sine(30°) = 1/2.
- In Quadrant III, both cosine and sine are negative: (-√3/2, -1/2).

8. Answer: -√2/2

- 315° is in Quadrant IV; the reference angle is 360° - 315° = 45°.
- From the 45-45-90 triangle, sine(45°) = √2/2.
- In Quadrant IV, sine is negative, so sine(315°) = -√2/2.

9. Answer: 1 (Pythagorean Identity)

- From the unit circle, cos(30°) = √3/2 and sin(30°) = 1/2.
- Compute: $(\sqrt{3}/2)^2 + (1/2)^2 = 3/4 + 1/4 = 1$.

- This demonstrates the Pythagorean Identity: $\cos^2(\theta) + \sin^2(\theta) = 1$ for all angles θ.

10. Answer: θ = 120°: sine = $\sqrt{3}/2$ (QII); θ = 240°: sine = $-\sqrt{3}/2$ (QIII)

- The reference angle for cos = 1/2 is 60° (from the 30-60-90 triangle).
- Cosine is negative in Quadrant II and Quadrant III.
- Quadrant II: θ = 180° - 60° = 120°. Here sine(120°) = $\sqrt{3}/2$.
- Quadrant III: θ = 180° + 60° = 240°. Here sine(240°) = $-\sqrt{3}/2$.
- Both points lie on the unit circle with x-coordinate -1/2.