De Moivre's Theorem: Powers of Complex Numbers

Precalculus / Trigonometry Worksheet · Grade 11–12

Name: ______________________________ Date: __________________

Learning Objectives

- Convert complex numbers from rectangular form to trigonometric (polar) form
- Apply De Moivre's Theorem to raise complex numbers to integer powers
- Evaluate trigonometric expressions using the unit circle and coterminal angles

Problems

1. Find the modulus r of the complex number shown below.

$$z = 3 + 4i$$

2. Find the argument theta (in degrees) of the complex number shown below. Round to the nearest degree if necessary.

$$z = 1 + \sqrt{3}\, i$$

3. Write the complex number shown below in trigonometric (polar) form z = r(cos θ + i sin θ).

$$z = -1 + i$$

4. Use De Moivre's Theorem to evaluate the expression below. Write your answer in rectangular form a + bi.

$$[2(\cos 30^\circ + i\sin 30^\circ)]^3$$

5. Use De Moivre's Theorem to evaluate the expression below. Write your answer in rectangular form a + bi.

$$[3(\cos 45^\circ + i\sin 45^\circ)]^4$$

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6. Convert the complex number shown below to trigonometric form, then use De Moivre's Theorem to compute the power. Write the final answer in rectangular form a + bi.

$$(1+i)^6$$

7. Compute the power shown below using De Moivre's Theorem. Write the final answer in rectangular form a + bi.

$$(-1+\sqrt{3}\,i)^5$$

8. Compute the power shown below using De Moivre's Theorem. Write the final answer in rectangular form a + bi.

$$(\sqrt{3}-i)^8$$

9. Use De Moivre's Theorem to compute the power shown below. Write the final answer in rectangular form a + bi.

$$(-1+\sqrt{3}\,i)^{12}$$

10. Use De Moivre's Theorem to compute the power shown below. Write the final answer in rectangular form a + bi.

$$(2+2i)^{10}$$

De Moivre's Theorem: Powers of Complex Numbers — Answer Key

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Answer Key

1. Answer: r = 5

- Use r = sqrt(a^2 + b^2) where a = 3 and b = 4
- r = sqrt(9 + 16) = sqrt(25) = 5

2. Answer: θ = 60°

- Use θ = tan■^{1}(b/a) = tan■^{1}(√3 / 1)
- tan■^{1}(√3) = 60°, and since a > 0 and b > 0 the angle is in Quadrant I, so θ = 60°

3. Answer: z = √2 (cos 135° + i sin 135°)

- r = sqrt((-1)^2 + 1^2) = sqrt(2)
- Reference angle: tan■^{1}(1/1) = 45°. Since a < 0 and b > 0, the angle is in Quadrant II: θ = 180° – 45° = 135°
- z = √2 (cos 135° + i sin 135°)

4. Answer: 8i (i.e. 0 + 8i)

- Apply De Moivre's: r^n = 2^3 = 8, and n·θ = 3 × 30° = 90°
- Result: 8(cos 90° + i sin 90°) = 8(0 + i·1) = 8i

5. Answer: -81 + 0i

- r^4 = 3^4 = 81, and n·θ = 4 × 45° = 180°
- 81(cos 180° + i sin 180°) = 81(-1 + 0i) = -81

6. Answer: -8 + 0i

- r = sqrt(1^2 + 1^2) = sqrt(2), θ = 45°
- Trig form: sqrt(2)(cos 45° + i sin 45°)
- Apply De Moivre's: r^6 = (sqrt(2))^6 = 8, n·θ = 6 × 45° = 270°
- 8(cos 270° + i sin 270°) = 8(0 – i) = -8i
- Wait — cos 270° = 0 and sin 270° = -1, so answer = 8(0 + i(-1)) = -8i

7. Answer: 16 + 16√3 i

- a = -1, b = √3; r = sqrt(1 + 3) = 2
- Reference angle: tan■^{1}(√3/1) = 60°. Since Quadrant II: θ = 120°
- Apply De Moivre's: r^5 = 2^5 = 32, n·θ = 5 × 120° = 600°
- Coterminal angle of 600°: 600° – 360° = 240°
- 32(cos 240° + i sin 240°) = 32(–1/2 + i(–√3/2)) = –16 – 16√3 i

8. Answer: 128 + (-128√3)i ≈ 128 – 128√3 i

- a = √3, b = -1; r = sqrt(3 + 1) = 2

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- θ = tan■¹(-1/√3) = -30° (Quadrant IV) = 330°
- r^8 = 2^8 = 256, n·θ = 8 × 330° = 2640°
- Coterminal: 2640° mod 360° = 2640 – 7×360 = 2640 – 2520 = 120°
- 256(cos 120° + i sin 120°) = 256(–1/2 + i√3/2) = –128 + 128√3 i

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9. Answer: 4096 + 0i

- r = 2, θ = 120° (from Quadrant II)
- r^12 = 2^12 = 4096, n·θ = 12 × 120° = 1440°
- Coterminal: 1440° – 4×360° = 1440° – 1440° = 0°
- 4096(cos 0° + i sin 0°) = 4096(1 + 0i) = 4096

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10. Answer: 0 – 32768i

- a = 2, b = 2; r = sqrt(4+4) = 2√2, θ = tan■¹(2/2) = 45°
- r^10 = (2√2)^10 = 2^10 · (√2)^10 = 1024 · 32 = 32768
- n·θ = 10 × 45° = 450°
- Coterminal: 450° – 360° = 90°
- 32768(cos 90° + i sin 90°) = 32768(0 + i·1) = 32768i

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