

Conic Sections: Parabolas

Analytic Geometry Worksheet · Grade 10–12

Name: _____

Date: _____

Learning Objectives

- Identify the vertex, focus, and directrix of a parabola from its equation or graph
- Write the standard equation of a parabola that opens vertically or horizontally
- Determine the direction a parabola opens based on the sign of p

Problems

1. A parabola is defined as the set of all points equidistant from a fixed point and a fixed line. What are the names of that fixed point and fixed line?

2. Identify which standard equation below represents a parabola opening vertically, and which represents one opening horizontally.

A: $(x - h)^2 = 4p(y - k)$ B: $(y - k)^2 = 4p(x - h)$

3. State the direction a parabola opens for each value of p described below.

(a) Vertical parabola, $p > 0$ (b) Vertical parabola, $p < 0$

4. A parabola has its vertex at the origin. Its focus is at the point shown below. Write the standard equation of the parabola.

Focus = $(0, 3)$

5. Find the vertex, focus, and directrix of the parabola whose equation is given below.

$(x - 2)^2 = 8(y - 1)$

6. Find the vertex, focus, and directrix of the parabola whose equation is given below.

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$$(y + 3)^2 = -12(x - 1)$$

7. Write the standard equation of the parabola with the vertex and directrix given below.

Vertex = $(4, -2)$, Directrix: $y = 1$

8. Convert the general form equation below to standard form, then identify the vertex and the value of p .

$$x^2 - 6x - 4y + 1 = 0$$

9. A parabola passes through the point given below, has its vertex at the origin, and opens to the right. Find its equation and the coordinates of the focus.

Point: $(4, 4)$

10. A spotlight mirror is shaped like a parabola. The bulb is placed at the focus, which is 5 cm from the vertex. The mirror opens upward. Write the equation of the parabola with vertex at the origin, then find the width of the mirror at a height of 5 cm above the vertex.

$$p = 5 \text{ cm}$$



Conic Sections: Parabolas — Answer Key

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Answer Key

1. Answer: The fixed point is the focus; the fixed line is the directrix.

- Recall the definition of a parabola.
 - The fixed point inside the parabola is called the focus.
 - The fixed line on the opposite side of the vertex is called the directrix.
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2. Answer: Equation A opens vertically; Equation B opens horizontally.

- When the squared term is $(x - h)^2$, the axis of symmetry is vertical, so the parabola opens up or down.
 - When the squared term is $(y - k)^2$, the axis of symmetry is horizontal, so the parabola opens left or right.
 - Therefore A is vertical and B is horizontal.
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3. Answer: (a) Opens upward; (b) Opens downward.

- For $(x-h)^2 = 4p(y-k)$: when p is positive the parabola opens upward.
 - When p is negative the parabola opens downward.
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4. Answer: $x^2 = 12y$

- The focus is at $(0, 3)$, so $h = 0$, $k = 0$, and $p = 3$.
 - Since the focus is above the vertex, the parabola opens upward.
 - Use $(x - h)^2 = 4p(y - k)$: $x^2 = 4(3)y = 12y$.
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5. Answer: Vertex $(2, 1)$; Focus $(2, 3)$; Directrix $y = -1$

- Compare with $(x - h)^2 = 4p(y - k)$: $h = 2$, $k = 1$, $4p = 8$ so $p = 2$.
 - Vertex = $(h, k) = (2, 1)$.
 - Focus = $(h, k + p) = (2, 1 + 2) = (2, 3)$.
 - Directrix: $y = k - p = 1 - 2 = -1$.
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6. Answer: Vertex $(1, -3)$; Focus $(-2, -3)$; Directrix $x = 4$

- Compare with $(y - k)^2 = 4p(x - h)$: $k = -3$, $h = 1$, $4p = -12$ so $p = -3$.
 - Since $p < 0$ and the equation is horizontal, the parabola opens to the left.
 - Vertex = $(1, -3)$.
 - Focus = $(h + p, k) = (1 + (-3), -3) = (-2, -3)$.
 - Directrix: $x = h - p = 1 - (-3) = 4$.
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7. Answer: $(x - 4)^2 = -12(y + 2)$

- The directrix is $y = 1$, which is above the vertex $y = -2$, so the parabola opens downward.
 - $p = k - (\text{directrix}) = -2 - 1 = -3$.
 - Use $(x - h)^2 = 4p(y - k)$: $(x - 4)^2 = 4(-3)(y - (-2)) = -12(y + 2)$.
-

8. Answer: $(x - 3)^2 = 4(y + 2)$; Vertex $(3, -2)$; $p = 1$

- Group x terms: $x^2 - 6x = 4y - 1$.

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- Complete the square: $(x - 3)^2 - 9 = 4y - 1$.
 - $(x - 3)^2 = 4y - 1 + 9 = 4y + 8 = 4(y + 2)$.
 - Standard form: $(x - 3)^2 = 4(1)(y + 2)$, so vertex = (3, -2) and $p = 1$.
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9. Answer: $y^2 = 4x$; Focus (1, 0)

- The parabola opens right with vertex at origin: $y^2 = 4px$.
 - Substitute the point (4, 4): $4^2 = 4p(4) \rightarrow 16 = 16p \rightarrow p = 1$.
 - Equation: $y^2 = 4x$.
 - Focus = $(p, 0) = (1, 0)$.
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10. Answer: Equation: $x^2 = 20y$; Width at $y = 5$ is 20 cm

- Parabola opens upward with vertex at origin: $x^2 = 4py = 4(5)y = 20y$.
 - To find width at height $y = 5$, substitute $y = 5$: $x^2 = 20(5) = 100$.
 - $x = \pm 10$, so the total width = $10 + 10 = 20$ cm.
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Scan to watch

